



Year 9

Term 1

Knowledge Organiser

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Year 9 Knowledge Organiser INDICES & STANDARD FORM

What do I need to be able to do?

- Use the laws of indices to simplify expressions.
- Convert between ordinary numbers and standard form and vice versa.
- Multiply and divide with standard form.
- Add and subtract with standard form.

Key Words

Square: A square number is the result of multiplying a number by itself.

Cube: A cube number is the result of multiplying a number by itself twice.

Root: A root is the reverse of a power.

Prime number: A prime is a number that has only two factors which are 1 and itself.

Indices: These are the squares, cubes and powers.

Standard form: This is a way of writing really big or really small numbers.

Ordinary numbers: These are numbers that are not yet in standard form. They are the normal form we see numbers in.

Laws of indices

Multiplication law: When multiplying with the same base (number/letter) we add the powers.

General rule: $a^m \times a^n = a^{m+n}$

$$2^5 \times 2^7 = 2^{5+7} = 2^{12}$$

$$x^3 \times x^8 = x^{3+8} = x^{11}$$

When multiplying the terms we add the powers together.

Division law: When dividing with the same base (number/letter) we subtract the powers.

General rule: $a^m \div a^n = a^{m-n}$

$$2^{14} \div 2^7 = 2^{14-7} = 2^7$$

$$x^{10} \div x^8 = x^{10-8} = x^2$$

When dividing the terms we subtract the powers together.

Brackets law: When raising a power to another power we multiply the powers together.

General rule: $(a^m)^n = a^{m \times n}$

$$(5^4)^2 = 5^{4 \times 2} = 5^8$$

$$(h^9)^3 = h^{9 \times 3} = h^{27}$$

When raising to a power we multiply the powers together.

Key facts: You need to also remember that:

$$p = p^1$$

$$p^0 = 1$$

Anything to the power zero is equal to 1.

Year 9 Knowledge Organiser INDICES & STANDARD FORM

Converting with standard form

Ordinary numbers: To change between ordinary numbers and standard form we need to use a power of 10.

$$120000 = 1.2 \times 10^5$$

This number need to be bigger than 1 and less than 10 to be in standard form.

$$0.005 = 5 \times 10^{-3}$$

Positive power = very big number.
Negative power = very small number.

Standard form: To change numbers from standard form back to ordinary numbers we multiply by the power of 10.

$$7.32 \times 10^4 = 73200$$

The power tells us how many places to move not how many zeros to add.

$$2.4 \times 10^{-3} = 0.0024$$

Multiplying standard form

Multiply standard form: We multiply the numbers and add the powers.

$$(5 \times 10^4) \times (7 \times 10^6)$$

$$= 35 \times 10^{10}$$

$$= 3.5 \times 10^{11}$$

This is not in standard form because 35 is not less than 10.

$$(3.2 \times 10^3) \times (4 \times 10^4)$$

$$= 12.8 \times 10^7$$

$$= 1.28 \times 10^8$$

Remember to add the powers together.

Dividing standard form

Divide standard form: We divide the numbers and subtract the powers.

$$(8 \times 10^9) \div (2 \times 10^6)$$

$$= 4 \times 10^3$$

This is already in standard form because 4 is less than 10.

$$(1.2 \times 10^5) \div (2 \times 10^2)$$

$$= 0.6 \times 10^3$$

$$= 6 \times 10^2$$

This is not in standard form because 0.6 is less than 1.

Adding and subtracting standard form

To add and subtract with standard form we must convert out of standard form into ordinary numbers first and then add/subtract.

$$(8.1 \times 10^5) + (2 \times 10^3)$$

Convert into ordinary numbers.

$$= 810000 + 2000$$

$$= 812000$$

$$= 8.12 \times 10^5$$

Add/subtract.

$$(2.35 \times 10^5) - (4.1 \times 10^3)$$

$$= 235000 - 4100$$

$$= 230900$$

$$= 2.309 \times 10^5$$

Change back into standard form.

Year 9 Knowledge Organiser NUMBER PROPERTIES

What do I need to be able to do?

- Recognise and calculate square numbers and roots.
- Recognise and calculate cube numbers and roots.
- Use BIDMAS to complete calculations.
- Identify factors and multiples.
- Identify a prime number and complete a prime factor tree.

Key Words

Square: A square number is the result of multiplying a number by itself.

Cube: A cube number is the result of multiplying a number by itself twice.

Root: A root is the reverse of a power.

Prime number: A prime is a number that has only two factors which are 1 and itself.

Multiple: A number in the given numbers times table.

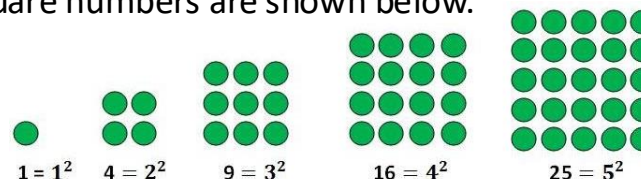
Factor: A number that fits into another number exactly.

Operation: In maths these are the functions $\times \div + -$.

Indices: These are the squares, cubes and powers.

Squares, cubes and roots

Square numbers: This is when we multiply a number by itself, the first 5 square numbers are shown below.



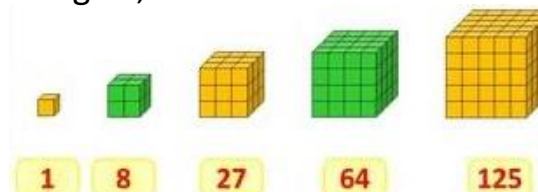
Square roots: This is the number that we started with to get the square numbers.

$$\sqrt{49} = 7 \text{ because } 7 \times 7 \text{ is } 49$$

$$\sqrt{100} = 10 \text{ because } 10 \times 10 \text{ is } 100$$

Remember the answer is 7 not 7x7.

Cube numbers: This is when we multiply a number by itself and then by itself again, the first 5 cube numbers are shown below.



Index form

Index number: An index number is a number which is raised to a power. The power, also known as the index, tells you how many times you have to multiply the number by itself.

2⁵ is the index notation.

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

Year 9 Knowledge Organiser NUMBER PROPERTIES

Multiples and factors

Multiples: The result of multiplying a number by an integer. It is the times table of a number.

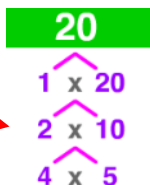
Multiples of 4: 4, 8, 12, 16, 20 ...

Multiples of 5: 5, 10, 15, 20, 25...

Multiples are the list of times tables.

Factors: A number that divides exactly into another number without a remainder. It is often helpful to write them in pairs.

Write them in pairs first so you don't miss any!



Factors of 20 = 1, 2, 4, 5, 10, 20

Prime numbers

Prime: This is a number that has exactly 2 factors; 1 and itself.

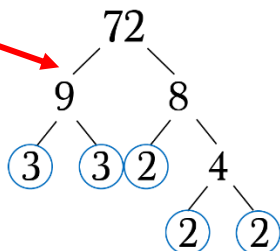
2 is the only even prime. The first 10 prime numbers are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

These are not the only prime numbers.

Prime factorisation: This is when we split a number into its prime factors using a factor tree. We circle the prime factors.

We need to find pairs of numbers that multiply to give the number above.



If a number is repeated we write it as a power.

$$72 = 2^3 \times 3^2$$

BIDMAS – order of operations

B Brackets

I Indices

D Division

M Multiplication

A Addition

S Subtraction

If a calculation contains the circled calculations then you need to work from left to right.

$$5 \times 4 - 8 \div 2$$

$$20 - 4 = 16$$

This question can be split into two separate calculations which are then combined to get the answer.

We need to deal with the powers inside the brackets first by calculating 2^2 .

$$(2^2 + 6)^2 \times 4 - 8$$

$$(4 + 6)^2 \times 4 - 8$$

$$(10)^2 \times 4 - 8$$

Once the bracket has been fully calculated we then look at the operations on the outside of the bracket.

$$100 \times 4 - 8$$

$$400 - 8 = 392$$

Year 9 Knowledge Organiser EXPRESSIONS & EQUATIONS

What do I need to be able to do?

- Factorise an expression into a single bracket.
- Solve one step equations.
- Solve two step equations.
- Solve equations with brackets.
- Solve equations with unknowns on both sides.

Key Words

Inverse: This is another word for opposite. We complete the opposite operation to the one shown in the question.

Expression: Shows a mathematical relationship whereby there is no solution.

Equation: A mathematical statement that shows that two expressions are equal.

Factorise: Putting an expression into brackets by finding the HCF.

Solve: To get the solution or answer to a question.

Factorising

Factorising: To factorise an expression we need to find the highest common factor of all of the terms in the expression.

Expression: $6x + 15$

The HCF of $6x$ and $15 = 3$

$$6x \div 3 = 2x$$

$$15 \div 3 = 5$$

$6x$ and 15 are both in the 3 times table.

The terms from dividing by the HCF.

Answer: $3(2x + 5)$

HCF

Expression: $4x^2 - 6x$

The HCF of $4x^2$ and $6x = 2x$

$$4x^2 \div 2x = 2x$$

$$6x \div 2x = 3$$

$4x^2$ and $6x$ are both in the 2 times table and both contain an x .

The terms from dividing by the HCF.

Answer: $2x(2x - 3)$

HCF

Solving one step equations

To solve any equation we need to do the inverse of the operation that we see.

$$t + 4 = 10$$

$$\begin{array}{cc} -4 & -4 \end{array}$$

$$t = 6$$

The inverse of add is subtract and vice versa.

$$c - 3 = 6$$

$$\begin{array}{cc} +3 & +3 \end{array}$$

$$c = 9$$

$$6y = 30$$

$$\begin{array}{cc} \div 6 & \div 6 \end{array}$$

$$y = 5$$

The inverse of multiply is divide and vice versa.

$$\frac{m}{7} = 4$$

$$\begin{array}{cc} \times 7 & \times 7 \end{array}$$

$$m = 28$$



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Year 9 Knowledge Organiser EXPRESSIONS & EQUATIONS

Solving two step equations

To solve a two step equation we need to complete 2 inverse calculations in a specific order.

$$6y + 2 = 32$$

$-2 \quad -2$

Subtract first because the 2 is separate from the x.

$$6y = 30$$

$\div 6 \quad \div 6$

$$y = 5$$

Divide because it is the inverse of multiplying.

$$\frac{w-5}{3} = 4$$

$\times 3 \quad \times 3$

Multiply first because the entire expression is divided by 3.

$$w - 5 = 12$$

$+5 \quad +5$

$$w = 17$$

Add because it is the inverse of subtracting.

Solving equations with brackets

We must expand the bracket first and then solve by doing the inverse of the operations.


$$3(2x + 5) = 39$$

Expand brackets first.

$$6x + 15 = 39$$

$-15 \quad -15$

The inverse of +15 is -15.

$$6x = 24$$

$\div 6 \quad \div 6$

$$x = 4$$

The inverse of $\times 6$ is $\div 6$.

Solving with unknowns on both sides

To solve an equation with unknowns on both sides we need to collect all of the same terms together, still by looking at the inverse.

$$5x - 20 = 3x + 4$$

$-3x \quad -3x$

We subtract 3x from both sides because it is the smaller term of x.

$$2x - 20 = 4$$

$+20 \quad +20$

$$2x = 24$$

$\div 2 \quad \div 2$

$$x = 12$$

Then solve like a normal two step equation.

$$2x - 10 = 5x + 2$$

$-2x \quad -2x$

We subtract 2x from both sides because it is the smaller term of x.

$$-10 = 3x + 2$$

$-2 \quad -2$

$$-12 = 3x$$

$\div 3 \quad \div 3$

$$-4 = x$$

Then solve like a normal two step equation.

Top tip: Always subtract/add the smaller number of terms to avoid getting a negative term at the end.

Year 9 Knowledge Organiser EXPRESSIONS & FORMULAE

What do I need to be able to do?

- Simplify expressions by collecting like terms.
- Write an expression from a given sentence.
- Expand a single bracket.
- Substitute into an expression or formula.
- Expand double brackets.
- Rearrange a formula.

Key Words

Formula: A rule written using symbols that describe a relationship between different quantities.

Expression: Shows a mathematical relationship whereby there is no solution.

Equation: A mathematical statement that shows that two expressions are equal.

Term: This is a number and a letter put together, for example $3x$ is a term.

Rearrange: Complete inverse operations to change the structure of the formula.

Expanding brackets

Each term inside the bracket it multiplied by this value.

$$\begin{aligned} & 2(3a - 4) \\ &= 6a - 8 \\ &= 6a - 8 \end{aligned}$$

Expanding: To expand a bracket you multiply each term on the inside of the bracket by the term on the outside of the bracket. It is very important that you multiply all of the terms.

Simplifying expressions

The operation symbol in front of the term tells you what to do.

Simplify:

$$\begin{aligned} & 4a + 3b - a + 2b \\ &= 3a + 5b \end{aligned}$$

Collecting like terms: You can only collect terms with the same letter together. The 'a' terms can only be collected with other 'a' terms. The operation symbol in front of the terms tells you what to do with it.

Expand and simplify:

$$\begin{aligned} & 2(4a + 2b) - 2(a + 3b) \\ &= 8a + 4b - 2a - 6b \\ &= 6a - 2b \end{aligned}$$

Expanding and collecting like terms: You need to first expand the brackets separately and then collect the terms with the same letter like you would in the previous question.

Year 9 Knowledge Organiser EXPRESSIONS & FORMULAE

Expanding double brackets

Expanding: To expand double brackets we need to use a grid to make sure that all of the terms have been multiplied together.

$$(x + 1)(x + 4)$$

×	x	+1
x	x^2	$+1x$
+4	$+4x$	$+4$

$$= x^2 + 1x + 4x + 4$$

$$= x^2 + 5x + 4$$

First bracket along the top and the second down the side.

Multiply all of the terms together.

Simplify the expression.

$$(2x + 1)(x - 3)$$

	2x	+	1
x	$2x^2$		$1x$
-3	$-6x$		-3

$$= 2x^2 + 1x - 6x - 3$$

$$= 2x^2 - 5x - 3$$

Substitution

Substitution: This is where we replace the letter we see for the number that it is worth.

Remember that 3y means 3 multiplied by the value of y.

For example: If $w = 6$ and $y = 5$

a) $3y - 2 = 3 \times 5 - 2 = 15 - 2 = 13$

b) $8w + 2y = 8 \times 6 + 2 \times 5 = 48 + 10 = 58$

c) $2w^2 - 7 = 2 \times 6^2 - 7 = 72 - 7 = 65$

Calculate the powers first then multiply by the number in front.

Rearranging formulae

To rearrange a formula we use the same method that we would use to solve an equation however, we do not get a solution.

To make y the subject of this formula:

$$6y + 2 = x$$

$$\begin{matrix} -2 & -2 \end{matrix}$$

Subtract first because the 2 is separate from the y.

$$6y = x - 2$$

$$\begin{matrix} \div 6 & \div 6 \end{matrix}$$

Divide because it is the inverse of multiplying.

$$y = \frac{x - 2}{6}$$

y is now the subject of the formula because it is $y =$ not $x =$.

To make x the subject of this formula:

$$6x - 2c = 3x + 4$$

$$\begin{matrix} -3x & -3x \end{matrix}$$

We subtract 3x from both sides because it is the smaller amount of x and there are x's on both sides.

$$3x - 2c = 4$$

$$\begin{matrix} +2c & +2c \end{matrix}$$

Add 2c to both sides because x needs to be on its own.

$$3x = 2c + 4$$

$$\begin{matrix} \div 3 & \div 3 \end{matrix}$$

$$x = \frac{2c + 4}{3}$$

x is now the subject of the formula because it is $x =$.

Year 9 Knowledge Organiser AVERAGES & RANGE

What do I need to be able to do?

- Calculate averages and range from a list.
- Explain the difference between discrete and continuous data.
- Calculate averages from a discrete table.
- Calculate averages from grouped data.
- Calculate the IQR from a set of data.

Key Words

Mean: Add all of the numbers together and divide by how many there were.

Range: The biggest number subtract the smallest number.

Mode: The value there the most times.

Median: The middle number when they are in order.

Frequency: The number which tells us how many pieces of data there are.

Discrete: Data that can only take certain values.

Continuous: Data that can take certain values in a given range.

IQR: Inter quartile range is the range for the middle 50% of the data.

Averages and range from a list

Here is a list of numbers: 12, 15, 10, 8, 15

Mean: The mean is the average of a set of data.

Add all of the values together $12 + 15 + 10 + 8 + 15 = 60$
Divide this by the number of values in your list $60 \div 5 = 12$
The mean of this data is 12.

Mode: This is the value that appears the most times in the list. So if we look at the list above then the mode would be 15 because 15 appears twice in the list.

Range: This is the difference between the biggest and smallest numbers in the list.
 $15 - 8 = 7$ so the range for the list above is 7.

Median: This is the middle number but only when the list is in order from smallest to biggest.

8 10 12 15 15

This is in the middle so it must be the median.

12 is in the middle of the list so the median must be 12.

Year 9 Knowledge Organiser AVERAGES & RANGE

Averages from a table

5 3 4 5 6 7 2 3 6 3 5 3 2 7 6 4 3 5 5 6 2 3 3 6 5

Score x	Tally	Frequency f	Freq $\times$ Score fx
2		3	6
3		7	21
4		2	8
5		6	30
6		5	30
7		2	14

To calculate the mean we need to add in an extra column for score $\times$ frequency.

$$\text{Mean} = \frac{\text{Total of freq } \times \text{ score}}{\text{Total freq}}$$

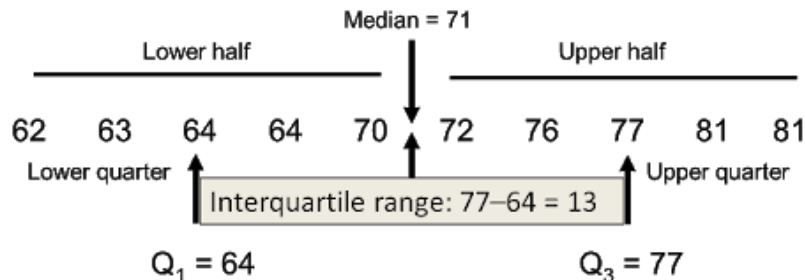
$$\text{Mean} = \frac{109}{25} = 4.36$$

Mode = 3 because the score of 3 has the highest frequency.

$$\text{Range} = 7 - 2 = 5$$

The range is always from the first column.

Interquartile range (IQR)



Calculate the median first. Then first the LQ and the UQ. The IQR is then the difference between these 2 values.

Averages from a table – Grouped data

To calculate the mean from a grouped frequency table we need to add on 2 columns, one for the midpoint and one for the frequency $\times$ midpoint.

number of laps	frequency	midpoint(x)	mp $\times$ f
1 - 5	2	3	6
6 - 10	9	8	72
11 - 15	15	13	195
16 - 20	20	18	360
21 - 25	17	23	391
26 - 30	25	28	700
31 - 35	2	33	66
36 - 40	1	38	38
	$\sum f = 91$		$\sum fx = 1828$

Calculate the midpoint of the group, $\frac{1+5}{2} = 3$.

Divide the total of the midpoint $\times$ freq column by the total of the freq column.

$$\text{Mean} = \frac{1828}{91} = 20.1$$

Mode = 26 - 30 because this group has the highest frequency.

Median = $\frac{91+1}{2} = 46$ (the 46th piece of data is the median. We need to find which group it is in. We start with the top frequency and keep adding the frequencies together until we get 46.

$2 + 9 + 15 + 20 = 46$ so the group 16 - 20 is the median.

Year 9 Knowledge Organiser REPRESENTING DATA

What do I need to be able to do?

- Read and draw a pie chart using a protractor.
- Draw a stem and leaf diagram.
- Draw a scatter graph.
- Identify outliers on a scatter graph.
- Draw a line of best fit on a scatter graph.
- Estimate values from a scatter graph.

Key Words

Frequency: The number which tells us how many pieces of data there are.

Protractor: This is a piece of equipment used to measure angles.

Scatter graph: This is a graph that shows the relationship between two variables.

Correlation: This describes if the data is increasing or decreasing.

Line of best fit: This is a line drawn on a scatter graph that follows the trend of the data.

Outlier: This is a data point that does not fit with the pattern of the rest of the data.

Pie charts

Pie charts represent discrete data. A circle is divided into segments, where each segment represents a data category. The size of each segment matches its proportion of the total amount.

Sport	Frequency	Angle
Swimming	12	$12 \times 15 = 180^\circ$
Netball	6	$6 \times 15 = 90^\circ$
Football	3	$3 \times 15 = 45^\circ$
Gymnastics	3	$3 \times 15 = 45^\circ$

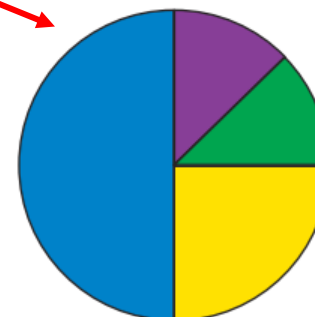
Total = 24

Each person: $360^\circ \div 24 = 15^\circ$

1. Find the total frequency.
2. Calculate one person by doing $360^\circ \div$ frequency.
3. Multiply each frequency by this value to get the angle size for each section.

You must use a protractor carefully to measure each section.

A pie chart to show children's favourite sports



Key

- swimming
- netball
- football
- gymnastics

You must label each section or use a key.

Year 9 Knowledge Organiser REPRESENTING DATA

Stem and leaf diagrams

Stem and leaf diagram: This is a plot where each data value is split into a "leaf" (usually the last digit) and a "stem" (the other digits). The leaf part is only ever 1 digit, the stem can be several.

Here is a list of numbers and the stem and leaf diagram:

68, 75, 77, 79, 80, 82, 92, 96, 96, 97

Stem	Leaf
6	8
7	5 7 9
8	0 2
9	2 6 6 7

The 'leaves' must be from smallest to biggest in each row.

Key 6|8 = 68

You must include a key to explain what the stem and leaf shows.

Mode = 96 because 96 appears twice.

Median = 81 because 81 is in the middle of 80 and 82.

Range = $97 - 68 = 29$.

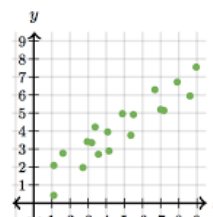
Mean = $\frac{68+75+77+79+80+82+92+96+96+97}{10} = 84.2$

We calculate these in the same way we would from a list.

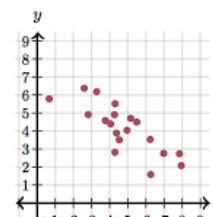
Scatter graphs

Scatter graph: This is a graph of plotted points that show the relationship between two sets of data.

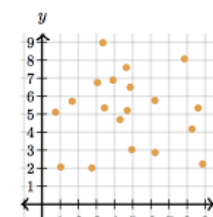
Types of correlation:



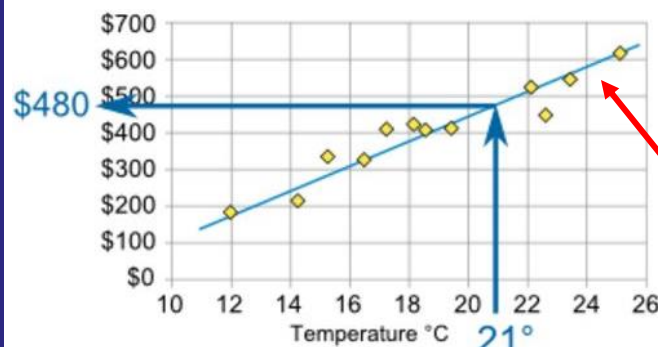
Positive



Negative



No correlation



The line of best fit needs to follow the trend of the data and have around half of the points either side. It does not need to start from 0.

Correlation: Positive correlation.

Description: As the temperature increases so does the amount of money made.

Estimate: $21^{\circ}\text{C} = \$480$.

Year 9 Knowledge Organiser REPRESENTING DATA

What do I need to be able to do?

- Complete a tally chart and design a data collection sheet.
- Complete and read information from a pictogram.
- Draw and read a bar chart and dual bar chart.
- Compare and interpret data in different contexts.
- Draw, read and compare box plots.

Key Words

Range: The biggest number subtract the smallest number.

Mode: The value there the most times.

Median: The middle number when they are in order.

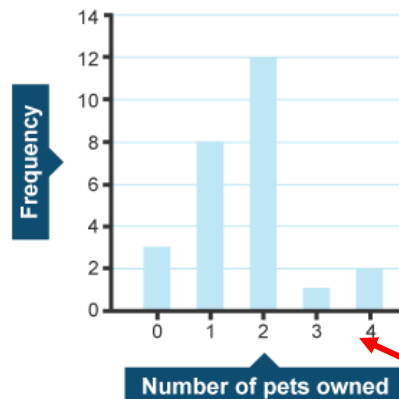
Frequency: The number which tells us how many pieces of data there are.

Bar chart: A diagram where the bars represent the frequency.

Box plot: A rectangular box that splits the data into 4 quartiles.

Interquartile range: This refers to the central 50% of data.

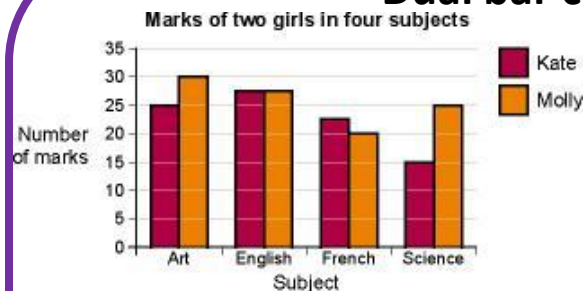
Bar Charts



A bar chart has a horizontal axis and a vertical axis. The x axis is for the type of data and the y axis shows the frequency. The bars show the data value of each category. There must be a gap between each bar and the scale must increase in the same sized intervals and the axes must be labelled.

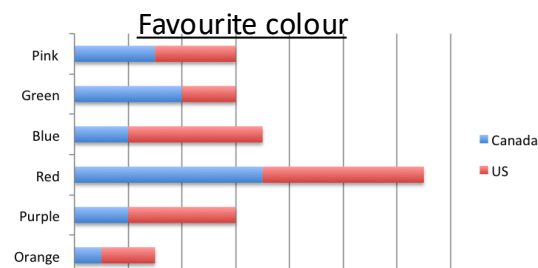
You must include gaps and labels.

Dual bar charts



You must include a key!

A dual bar chart is has most of the same features of a single bar chart. The bars show the data value of each category but this time there are 2 variables, usually 2 people of male/female. You must include a key.

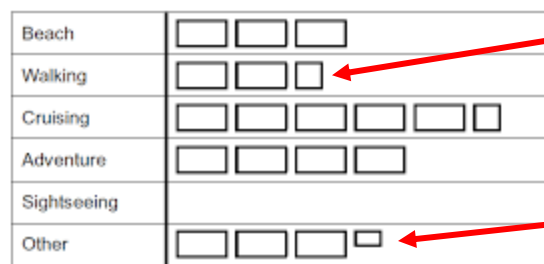


Year 9 Knowledge Organiser REPRESENTING DATA

Pictograms

Pictograms are similar to bar charts, but the data is shown in pictures. A pictogram must have a key so that you know what a full image represents.

Looking at this diagram:



This represents 2 people because it is half of the diagram in the key.

This represents 1 person because it is a quarter of the diagram in the key.

Key : represents 4 people.

Beach would show $4 + 4 + 4 = 12$ people.

Walking would show $4 + 4 + 2 = 10$ people.

Sightseeing has a total of 7 people and would be represented as:



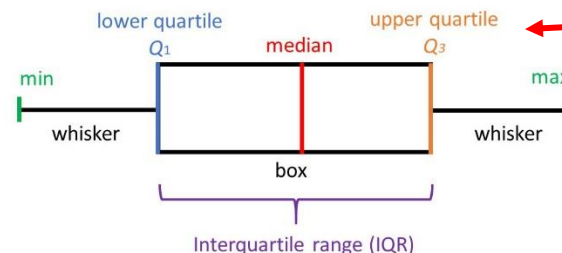
Interpreting data

Information can be shown in tables, charts or graphs.

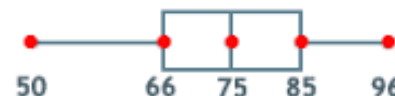
Interpreting data simply means understanding or working out what is being shown by a table, graph or chart and being able to answer questions about that information.

Box Plots

Box plot: This is a graphic way to display the median, quartiles, and extremes of a data set on a number line to show the distribution of the data.



Sometimes we won't be given all of this, we may be given the range and have to use this to find the min or max.



To find the IQR: $UQ - LQ$.

Data = 50, 60, 66, 70, 75, 80, 85, 89, 96
 Minimum = 50
 Lower Quartile = 66
 Median = 75
 Upper Quartile = 85
 Maximum = 96

$$IQR = 85 - 66 = 19$$

Comparing box plots: You must make sure that you compare:

- The median values
- The IQR or the range

One of these points must link back to the context of the question.

Year 9 Knowledge Organiser FDP

What do I need to be able to do?

- Convert between fractions, decimals and percentages.
- Calculate a percentage of an amount.
- Use a multiplier to calculate a percentage of an amount.
- Calculate a percentage increase.
- Calculate a percentage decrease.

Key Words

Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).

Integer: Whole number.

Ascending Order: Place in order, smallest to largest.

Descending Order: Place in order, largest to smallest.

Percentage: Out of one hundred.

Decimal: A decimal is a fraction written in a special form e.g. 0.6.

Multiplier: This is used to calculate percentages when we have a calculator.

Increase: When an amount goes up.

Decrease: When an amount goes down.

Fractions, decimals and percentages

You need to be able to convert between fractions, decimals and percentages.

Percentages:

$$30\% = 0.30 \text{ or } 0.3 = \frac{30}{100} \text{ or } \frac{3}{10}$$

$$8\% = 0.08 = \frac{8}{100} \text{ or } \frac{4}{50} \text{ or } \frac{2}{25}$$

We sometimes have to then simplify our fraction using common times tables.

Percent means out of 100 so this is why we start with 100 as the denominator.

Decimals:

$$0.15 = 15\% = \frac{15}{100} \text{ or } \frac{3}{20}$$

$$0.02 = 2\% = \frac{2}{100} \text{ or } \frac{1}{50}$$

If the decimal has 0 tenths then it is less than 10%.

Fractions:

$$\frac{45}{100} = 0.45 = 45\%$$

$$\frac{12}{50} \text{ or } \frac{24}{100} = 0.24 = 24\%$$

Unless we know the answer we must make the denominator 100 then convert.

These are some of the conversions that you need to learn.

Top tips - To convert:

- Percentages to decimals divide by 100.
- Decimals to percentages multiply by 100.
- Percentages to fractions, put over 100.
- Fractions make sure the denominator is 100.

F	D	P
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%



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Year 9 Knowledge Organiser FDP

Percentage of an amount – Non calculator

To calculate any percentage it is useful to start with 10%.

30% of 120: $10\% = 120 \div 10 = 12$ To find 10% we divide by 10.
 $30\% = 3 \times 12 = 36$ To find 30% we multiply 10% by 3.

45% of 80: $10\% = 80 \div 10 = 8$ $5\% = 8 \div 2 = 4$
 $40\% = 4 \times 8 = 32$
 $45\% = 40\% + 5\% = 32 + 4 = 36$ 5% is half of 10% so we divide by 2.

To find 1% we divide the starting amount by 100.

$1\% \text{ of } 30 = 30 \div 100 = 0.3.$

Percentage of an amount – Calculator

When we have a calculator we can use a multiplier; this is the decimal equivalent of the percentage.

80% of 120: $80\% = 0.80$
 $80\% \text{ of } 120 = 0.80 \times 120 = 96$ Change the percentage to a decimal and then multiply.

33% of 90: $33\% = 0.33$
 $33\% \text{ of } 90 = 0.33 \times 90 = 29.7$ Be careful if the percentage is less than 10.

4% of 88: $4\% = 0.04$
 $4\% \text{ of } 88 = 0.04 \times 88 = 3.52$ Take care using decimal percentages, still divide by 100.

12.5% of 42: $12.5\% = 0.125$
 $12.5\% \text{ of } 42 = 0.125 \times 42 = 5.25$

Percentage increase and decrease

Increase: To calculate a percentage increase we calculate the percentage and add the value on to the original amount.

Non Calculator: Increase 70 by 65%

$10\% = 70 \div 10 = 7$ $5\% = 7 \div 2 = 3.5$

$60\% = 6 \times 7 = 42$

$65\% = 60\% + 5\% = 42 + 3.5 = 45.5$ Calculate 65% by splitting into 10% and 5% and then add the answer on to the original amount.

$70 + 45.5 = 115.5$

Calculator: Increase 130 by 26%

Calculate 26% using a multiplier and add this answer onto the original amount.

$26\% \text{ of } 130 = 0.26 \times 130 = 33.8$

$130 + 33.8 = 163.8$

Decrease: To calculate a percentage decrease we calculate the percentage and subtract the value off the original amount.

Non Calculator: Decrease 20 by 35%

$10\% = 20 \div 10 = 2$ $5\% = 2 \div 2 = 1$

$30\% = 3 \times 2 = 6$

$35\% = 30\% + 5\% = 6 + 1 = 7$ Calculate 35% by splitting into 10% and 5% and then subtract the answer off the original amount.

$20 - 7 = 13$

Calculator: Decrease 65 by 14%

Calculate 14% using a multiplier and subtract this answer off the original amount.

$14\% \text{ of } 65 = 0.14 \times 65 = 9.1$

$65 - 9.1 = 55.9$

Year 9 Knowledge Organiser MULTIPLICATIVE REASONING

What do I need to be able to do?

- Enlarge a shape using a scale factor.
- Enlarge a shape with a fractional or negative scale factor.
- Calculate with compound interest.
- Calculate with reverse percentages to find the original value.
- Solve direct/inverse proportion problems.

Key Words

Enlargement: A change in size, either bigger or smaller.

Scale factor: This is how much the shape increases or decreases by.

Percentage: Out of one hundred.

Decimal: A decimal is a fraction written in a special form e.g. 0.6.

Multiplier: This is used to calculate percentages when we have a calculator.

Compound interest: Calculating interest where interest earned over time is added to the original value.

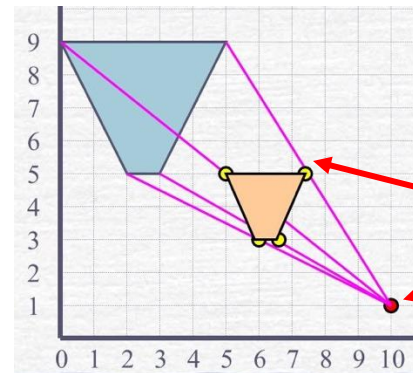
Direct proportion: Where both variables increase.

Inverse proportion: One variable increases and the other decreases.

Enlargement

An enlargement is when a shape changes in size by using a scale factor. The scale factor can make a shape bigger or smaller. A scale factor of 2 = shape doubles in size, a scale factor of $\frac{1}{2}$ would halve the size.

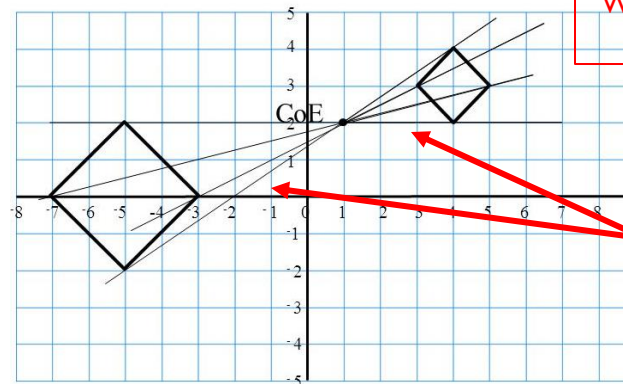
Fractional enlargements: Enlargement SF $\frac{1}{2}$ with centre (10,1).



Depending on the fraction the shape could be made bigger or smaller.

The shape is half the size. We divided the distance from the centre to the original shape by 2.

Negative enlargements: Enlargement SF -2 with centre (1, 2).



We can also have scale factors that are negative fractions.

The shape is twice the size but because it is a negative scale factor, we measure the increased difference from the centre in the opposite direction.

Year 9 Knowledge Organiser MULTIPLICATIVE REASONING

Compound interest

Compound Interest means that you work out the interest for the first period, add it to the total, and then calculate the interest for the next period etc.

Below is a formula we can use to make the calculation quicker.

$$\text{Amount of money after } x \text{ years} = \text{amount} \times \text{multiplier}^x$$

Tess invested £5000 at 4% compound interest for five years. How much was the investment worth after five years?

$$5000 \times 1.04^5 = £6083.26$$

Remember to round money to 2dp.

$$100\% + 4\% = 104\% = 1.04$$

Reverse percentages

Reverse percentages are used when the percentage and the final number is given, and the original number needs to be found.

Q: A shop offers 30% off in a sale. The sale price of a pair of shoes is £84. Calculate the cost of the shoes before the sale.

£84 = 70% of the original amount.

$$\text{A: } \frac{84}{70} \times 100 = £120$$

$$\frac{\text{Value}}{\text{Equivalent percentage}} \times 100 = \text{Original amount}$$

Q: After a 4% wage rise, Bill earned £1248 each month. Calculate how much Bill earned each month before the wage rise.

$$\text{A: } \frac{1248}{104} \times 100 = £1200$$

£1248 = 104% of the original amount.

Direct and inverse proportion

Direct: Where both variables increase.

y is directly proportional to x, and when x=3 then y=15. Calculate y when x = 6.

$$y \propto x$$

$$y = kx$$

These are the general rules.

$$15 = k \times 3$$

$$5 = k$$

Sub in the values you know and complete the inverse to calculate k (15 ÷ 3 = 5).

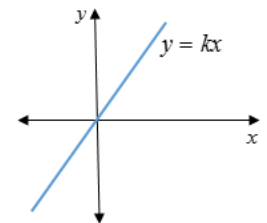
$$y = 5x$$

$$y = 5 \times 6 = 30$$

This is the formula for the given question.

$$y \propto x$$

$$y = kx \text{ for a constant } k$$



Inverse: Where one variable increases and the other decreases.

y is inversely proportional to x. When y=2, x=3. Work out the value of y when x = 12.

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

These are the general rules.

$$2 = \frac{k}{3}$$

$$6 = k$$

Sub in the values you know and complete the inverse to calculate k (2 × 3 = 6).

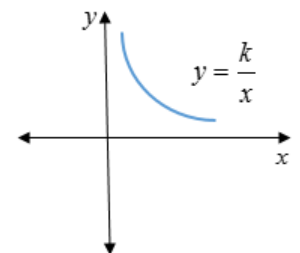
$$y = \frac{6}{x}$$

$$y = \frac{6}{12} = 0.5$$

This is the formula for the given question.

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x} \text{ for a constant } k$$





Year 9

Term 2

Knowledge Organiser

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Year 9 Knowledge Organiser CONSTRUCTIONS

What do I need to be able to do?

- Bisect a line using a compass.
- Bisect an angle using a compass.
- Construct SAS and ASA triangles with a protractor.
- Construct an SSS triangle with a compass.
- Construct the locus of a point.

Key Words

Construction: To draw a shape, line or angle accurately using a compass and ruler.

Loci: Set of points with the same rule.

Vertex: Corner of a shape or angle.

Perpendicular: Two lines that intersect at 90° .

Bisect: Divide into two parts.

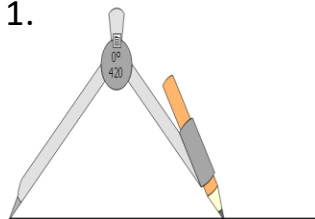
Equidistant: Equal distance.

Protractor: Used to measure and draw angles.

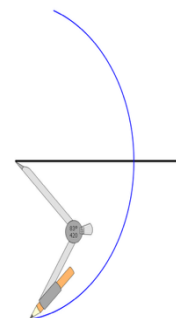
Compass: Used to bisect line and angles and complete constructions.

Bisect a line

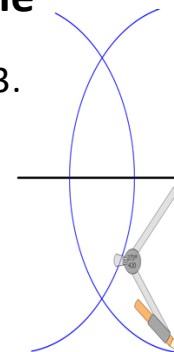
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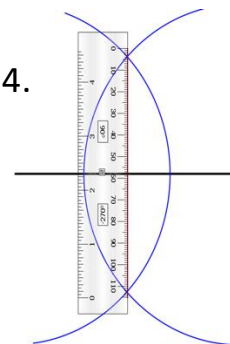
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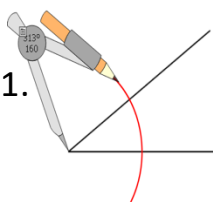
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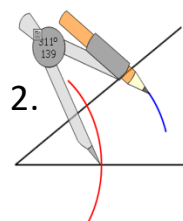
1. Place the spike of the compass at one end of line and open more than halfway along the line.
2. Draw an arc above the line and below your line.
3. Keeping the same compass width, draw an arc from other end of line.
4. Place ruler where the arcs cross and draw the bisecting line.

Bisect an angle

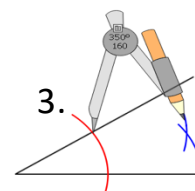
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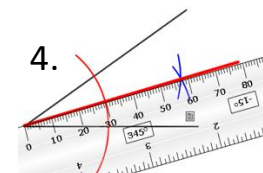
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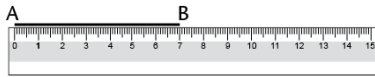
1. Place the compass on the vertex (corner) of the angle. Draw an arc across each arm of the angle.
2. Place the compass on the point where one arc crosses an arm and draw an arc inside the angle.
3. Without changing the compass width, repeat for the other arm so that the two arcs cross.
4. Use a ruler to join the vertex to the point where the arcs cross.

Year 9 Knowledge Organiser CONSTRUCTIONS

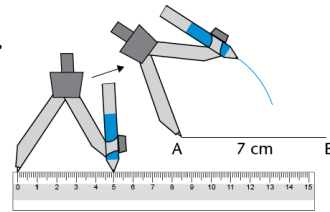
Constructing a triangle with a compass

Construct a triangle with sides of length 7cm, 5cm and 3cm.

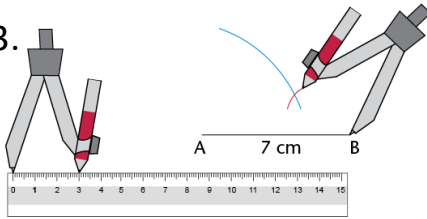
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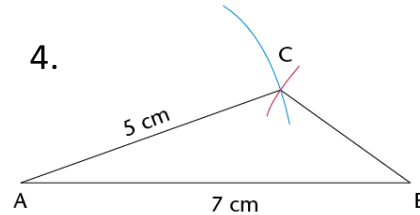
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3.



4.



1. Draw one side of the triangle using a ruler. It is often easier to start with the longest side.

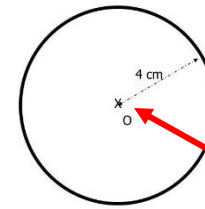
2. Set the compass width to 5 cm. Draw an arc 5 cm away from point A.

3. Set the compass width to 3 cm. Draw an arc from point B. This should cross your first arc.

4. Use your ruler to join points A and B to the point where the arcs intersect (C).

Loci

A locus is a path formed by a point which moves according to a rule. The plural is loci.



To find the locus 4cm from a specific point:

1. Open our compass to 4cm.
2. Put the spike on O and draw a circle.

Put the spike of the compass here.

If a point needs to be equidistant between 2 given places then we need to construct a line bisector.

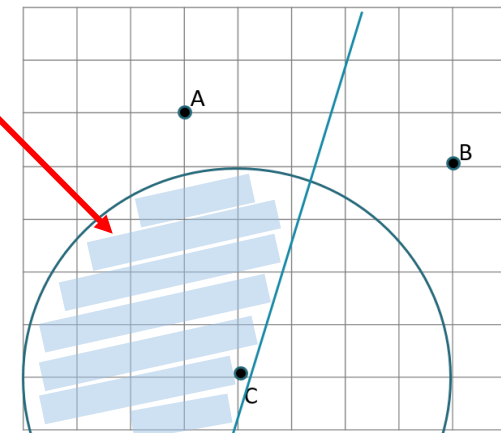
Shade the region that is:

- closer to A than B
- less than 4 cm from C

Construct a line bisector.

Draw a circle with a radius of 4cm.

This is shaded because it is inside the circle and closer to A using the line bisector.



Year 9 Knowledge Organiser EQUATIONS & INEQUALITIES

What do I need to be able to do?

- Read an inequality.
- Represent an inequality on a number line.
- Solve one step equations and inequalities.
- Solve two step equations and inequalities.
- Solve equations and inequalities with brackets.
- Solve equations and inequalities with unknowns on both sides.

Key Words

Inverse: This is another word for opposite. We complete the opposite operation to the one shown in the question.

Integer: A whole number.

Equation: A mathematical statement that shows that two expressions are equal.

Inequality: They compare two values, showing if one is less than or greater than another value.

Solve: To get the solution or answer to a question.

Inequalities

The list of integers for $-2 < x \leq 1$ is -1, 0, 1.

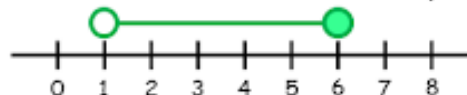
Check the symbols carefully, if they have the line underneath they include the end value.

○ Greater than $>$ Greater than or equal to $\geq$ ●

○ Less than $<$ Less than or equal to $\leq$ ●

Not equal to $\neq$

If x is **between** two values, use **two circles**:

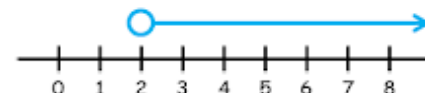


$$1 < x \leq 6$$

x is greater than 1, but less than or equal to 6.

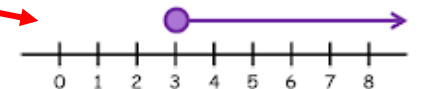
An **open circle** means that the value is **not included**:

$$x > 2 \quad x \text{ is greater than } 2$$



A **filled in circle** means that the value is **included**:

$$x \geq 3 \quad x \text{ is greater than or equal to } 3$$



Solving one step equations/inequalities

To solve any equation or inequality we need to do the inverse of the operation that we see.

$$t + 4 = 10$$

$$-4 \quad -4$$

$$t = 6$$

The inverse of add is subtract and vice versa.

$$c - 3 > 6$$

$$+3 \quad +3$$

$$c > 9$$

$$6y < 30$$

$$\div 6 \quad \div 6$$

$$y < 5$$

The inverse of multiply is divide and vice versa.

$$\frac{m}{7} = 4$$

$$\times 7 \quad \times 7$$

$$m = 28$$



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Year 9 Knowledge Organiser EQUATIONS & INEQUALITIES

Solving two step equations/inequalities

To solve a two step equation or inequality we need to complete 2 inverse calculations in a specific order.

$$6y + 2 = 32$$

$-2 \quad -2$

Subtract first because the 2 is separate from the y.

$$6y = 30$$

$\div 6 \quad \div 6$

Divide because it is the inverse of multiplying.

$$y = 5$$

$$\frac{w-5}{3} \geq 4$$

$\times 3 \quad \times 3$

Multiply first because the entire expression is divided by 3.

$$w - 5 \geq 12$$

$+5 \quad +5$

Add because it is the inverse of subtracting.

$$w \geq 17$$

Solving equations with brackets

We must expand the bracket first and then solve by doing the inverse of the operations. We use the same method for inequalities.


$$3(2x + 5) = 39$$

Expand brackets first.

$$6x + 15 = 39$$

$-15 \quad -15$

The inverse of +15 is -15.

$$6x = 24$$

$\div 6 \quad \div 6$

The inverse of $\times 6$ is $\div 6$.

$$x = 4$$

Solving with unknowns on both sides

To solve an equation or inequality with unknowns on both sides we need to collect all of the same terms together, still by looking at the inverse.

$$5x - 20 \leq 3x + 4$$

$-3x \quad -3x$

We subtract 3x from both sides because it is the smaller term of x.

$$2x - 20 \leq 4$$

$+20 \quad +20$

Then solve like a normal two step equation.

$$2x \leq 24$$

$\div 2 \quad \div 2$

$$x \leq 12$$

$$2x - 10 = 5x + 2$$

$-2x \quad -2x$

We subtract 2x from both sides because it is the smaller term of x.

$$-10 = 3x + 2$$

$-2 \quad -2$

$$-12 = 3x$$

$\div 3 \quad \div 3$

Then solve like a normal two step equation.

$$-4 = x$$

Top tip: Always subtract/add the smaller number of terms to avoid getting a negative term at the end.

Year 9 Knowledge Organiser SEQUENCES

What do I need to be able to do?

- Continue a sequence by finding the next term.
- Explain the rule of a sequence verbally and as a written explanation.
- Draw the next diagram in a sequence.
- Find the n th term of a sequence.
- Generate a sequence from the n th term.
- Recognise non linear sequences.

Key Words

Term: Each value in a sequence is called a term.

Rule: The value that a sequence increases or decreases by.

Sequence: A number or picture pattern with a specific rule.

Linear sequence: A sequence that increases or decreases by the same number between each term.

Nth term: A rule which allows you to calculate the term that is in the n th position of the sequence. Also known as the 'position to term' rule.

Generate: When we substitute values into the n th term to calculate the original sequence.

Finding the next term - numbers

When you need to find the next term in the sequence you need to work out what the general rule for the sequence is.

The rule is add 4 because the difference between each number is 4.

1, 5, 9, 13, 17
 $+4 \quad +4 \quad +4$

17 is the next number because $13 + 4 = 17$.

The rule is subtract 7 because the difference between each number is 7.

14, 7, 0, -7, -14
 $-7 \quad -7 \quad -7$

-14 is the next number because $-7 - 7 = -14$.

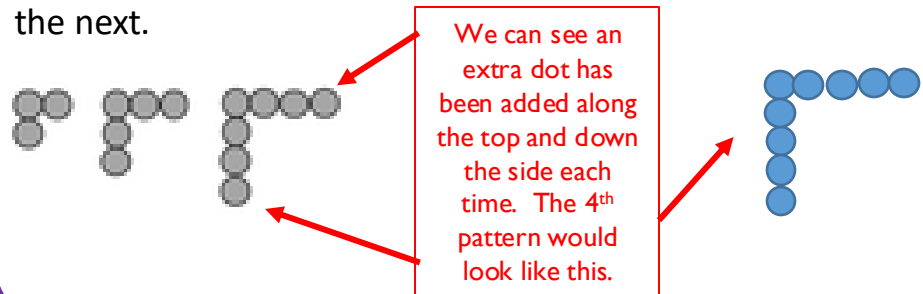
The rule is multiply by 2 because the numbers are doubling.

5, 10, 20, 40, 80
 $\times 2 \quad \times 2 \quad \times 2$

80 is the next number because $40 \times 2 = 80$

Finding the next term - diagrams

This is very similar to continuing a sequence of numbers, we just need to work out what has changed to get from one diagram to the next.



Year 9 Knowledge Organiser SEQUENCES

Finding the nth term

The nth term is the general rule for a sequence. We can use the nth term to then calculate any term in the sequence.

Here is a sequence: 5, 8, 11, 14, ...

Find the difference between the numbers.

5, 8, 11, 14
 $+3 \quad +3 \quad +3 = 3n$

This means that the nth term starts with $3n$ and we need to look at the 3 times table.

Remember to calculate how we get from the times table to the original sequence.

3, 6, 9, 12, ...
 5, 8, 11, 14, ...
 $+2$

The nth term is $3n + 2$.

Generating a sequence

To generate a sequence you substitute into the nth term. To get the first 3 terms of the sequence you would use 1, 2 & 3.

nth term = $3n - 1$

Substitute 1, 2 & 3 where n is in the nth term to get the first 3 numbers in the sequence.

n	$3n - 1$
1	$3 \times 1 - 1 = 2$
2	$3 \times 2 - 1 = 5$
3	$3 \times 3 - 1 = 8$

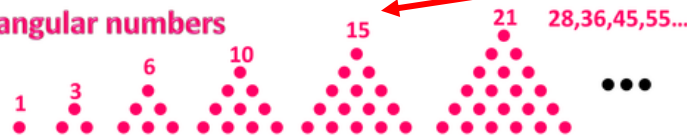
To get the 10th term:
 $3 \times 10 - 1 = 29$.

Sequence = 2, 5, 8,

Special sequences

Sometimes sequences do not increase or decrease by a consistent number. These can be quadratic sequences which include an n^2 term or they can be other special sequences some of which are shown below,

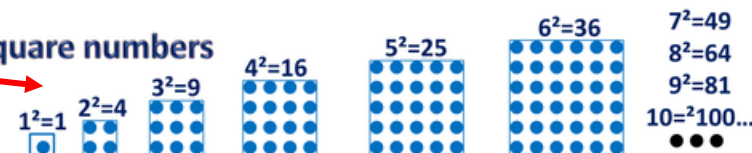
Triangular numbers



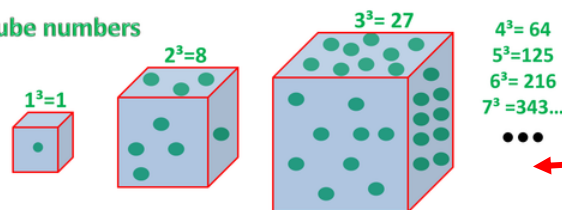
To get from one term to the next you can see that the difference increases by 1 more each time so 2, 3, 4, 5 etc.

These are the square numbers written as a sequence so 1×1 , 2×2 , 3×3 etc..

Square numbers



Cube numbers



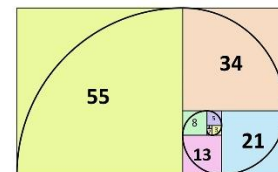
These are the cube numbers written as a sequence so $1 \times 1 \times 1$, $2 \times 2 \times 2$, $3 \times 3 \times 3$ etc..

The Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377 ...

$1+1=2$
 $1+2=3$
 $2+3=5$
 $3+5=8$
 $5+8=13$
 $8+13=21$
 $13+21=34$
 $21+34=55$
 $34+55=89$
 $55+89=144$
 $89+144=233$
 $144+233=377$

The Fibonacci sequence is when we add the second number in the sum to the answer to get the next term.



Year 9 Knowledge Organiser NON LINEAR SEQUENCES

What do I need to be able to do?

- Recognise the sequences of square, cube and triangular numbers.
- Explain the rule of a non linear sequence.
- Find the next term in a non linear sequence.
- Generate a quadratic sequence from the nth term.
- Find the nth term of a quadratic sequence.

Key Words

Term: Each value in a sequence is called a term.

Rule: The value that a sequence increases or decreases by.

Sequence: A number or picture pattern with a specific rule.

Nth term: A rule which allows you to calculate the term that is in the nth position of the sequence. Also known as the 'position to term' rule.

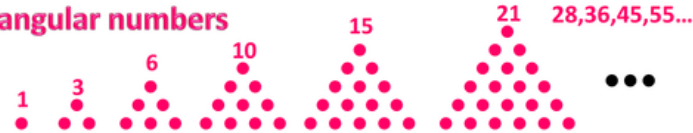
Generate: When we substitute values into the nth term to calculate the original sequence.

Quadratic sequence: Where n^2 is the highest power of n .

Special sequences

Sometimes sequences do not increase or decrease by a consistent number. These can be quadratic sequences which include an n^2 term or they can be other special sequences some of which are shown below.

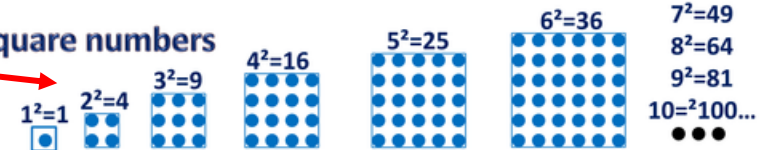
Triangular numbers



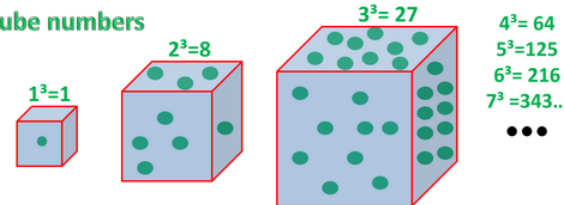
To get from one term to the next you can see that the difference increases by 1 more each time so 2, 3, 4, 5 etc.

These are the square numbers written as a sequence so 1×1 , 2×2 , 3×3 etc..

Square numbers



Cube numbers



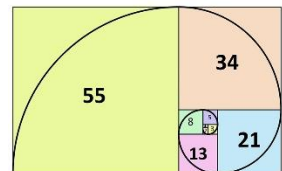
These are the cube numbers written as a sequence so $1 \times 1 \times 1$, $2 \times 2 \times 2$, $3 \times 3 \times 3$ etc..

The Fibonacci sequence is when we add the second number in the sum to the answer to get the next term.

The Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377...

$1+1=2$	$13+21=34$
$1+2=3$	$21+34=55$
$2+3=5$	$34+55=89$
$3+5=8$	$55+89=144$
$5+8=13$	$89+144=233$
$8+13=21$	$144+233=377$



Year 9 Knowledge Organiser NON LINEAR SEQUENCES

Finding the next term - numbers

When you need to find the next term in the sequence you need to find the rule.

The rule is add an extra 1 each time because the difference goes 1, 2, 3 etc.

1, 2, 4, 7, ... 11
+1 +2 +3

11 is the next number because $7 + 4 = 11$.

The rule is subtract 14, add 14 and this alternates each time.

7, -7, 7, -7, ... 7
-14 +14 -14

7 is the next number because $-7 + 14 = 7$.

The rule is half of the previous difference.

100, 80, 70, 65, .. 62.5
-20 -10 -5

62.5 is the next number because $65 - 2.5 = 62.5$.

Generating a quadratic sequence

To generate a sequence you substitute into the nth term. To get the first 3 terms of the sequence you would use 1, 2 & 3.

nth term = $n^2 + 2n$

Substitute 1, 2 & 3 where n is in the nth term to get the first 3 numbers in the sequence.

n	$n^2 + 2n$
1	$1^2 + 2 \times 1 = 3$
2	$2^2 + 2 \times 2 = 8$
3	$3^2 + 2 \times 3 = 15$

Sequence = 3, 8, 15...

To get the 10th term:
 $10^2 + 2 \times 10 = 120$.

We know this is non linear as the difference between each term is not the same.

Finding the nth term of a quadratic sequence

We know that a sequence is a quadratic sequence if the second difference is constant.

The nth term of the quadratic sequence will be of the form:

$$an^2 + bn + c$$

$$\begin{aligned} 2a &= 2^{\text{nd}} \text{ difference} \\ 3a + b &= 2^{\text{nd}} \text{ term} - 1^{\text{st}} \text{ term} \\ a + b + c &= 1^{\text{st}} \text{ term} \end{aligned}$$

We use these rules to calculate the values of a, b and c which then help use write the nth term.

2, 9, 18, 29, 42
+7 +11 +13 +15
+2 +2 +2

This is the second difference, they are all 2's so it must be a quadratic.

As $a=1$ there is $1n^2$ in the nth term, we do not need to write the 1.

$$\begin{aligned} 2a &= 2 \\ a &= 1 \end{aligned} \quad n^2$$

$$\begin{aligned} 3a + b &= 9 - 2 \\ 3 \times 1 + b &= 7 \\ 3 + b &= 7 \\ b &= 4 \end{aligned} \quad 4n$$

Substitute the value of a into the expression and rearrange to find b.

Substitute the values of a and b into the expression and rearrange to find c.

$$\begin{aligned} a + b + c &= 2 \\ 1 + 4 + c &= 2 \\ 5 + c &= 2 \\ c &= -3 \end{aligned} \quad -3$$

Substitute all 3 values into the general formula to make the nth term.

$$\text{nth term} = n^2 + 4n - 3$$

Year 9 Knowledge Organiser CIRCLES & PRISMS

What do I need to be able to do?

- Calculate the area of a circle.
- Calculate the circumference of a circle.
- Calculate the area/perimeter of a semi circle.
- Explain what volume means.
- Calculate the volume of a cube or cuboid.
- Calculate the volume of a triangular prism.
- Calculate the volume of a cylinder.

Key Words

Volume: The amount of space that an object occupies.

Cuboid: 3D shape with 6 square/rectangular faces.

Prism: A prism is a solid object with identical ends, flat faces and the same cross section all along its length.

Cross section: A cross section is the shape made by cutting straight across an object.

Circumference: The distance around the outside of the circle.

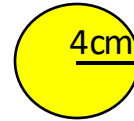
Semi-circle: Half a circle.

Perimeter: The distance around the shape.

Area of a circle

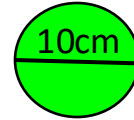
Formula:

Area of a circle = πr^2 (r is the radius)



$$\text{Area} = \pi \times 4^2 = 16\pi = 50.3\text{cm}^2$$

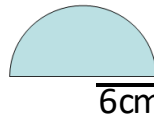
We need to calculate the radius first.



$$\text{Radius} = 10 \div 2 = 5\text{cm}$$

$$\text{Area} = \pi \times 5^2 = 25\pi = 78.5\text{cm}^2$$

Semi circle is half a circle so we need to divide by 2 at the end.



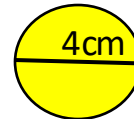
$$\text{Area of the full circle} = \pi \times 6^2 = 36\pi = 113.1\text{cm}^2$$

$$\text{Semi circle} = 36\pi \div 2 = 18\pi = 56.5\text{cm}^2$$

Circumference of a circle

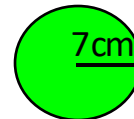
Formula:

Circumference of a circle = πd (d is the diameter)



$$\text{Circumference} = \pi \times 4 = 4\pi = 12.6\text{cm}$$

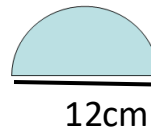
We need to calculate the diameter first.



$$\text{Diameter} = 7 \times 2 = 14\text{cm}$$

$$\text{Circumference} = \pi \times 14 = 14\pi = 44.0\text{cm}$$

Calculate the length of the curve then add in the diameter to find the perimeter.



$$\text{Full circle} = \pi \times 12 = 12\pi = 37.7\text{cm}$$

$$\text{Semi circle curve} = 37.7 \div 2 = 18.85\text{cm}$$

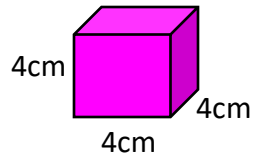
$$\text{Perimeter} = 18.85 + 12 = 30.85\text{cm}$$

Year 9 Knowledge Organiser CIRCLES & PRISMS

Volume - Cubes and cuboids

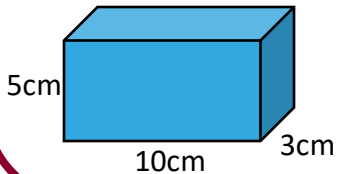
Cubes and cuboids: To calculate the volume of a cube and cuboid we use the following formula:

$$\text{Volume} = \text{Length} \times \text{Width} \times \text{Height}$$



$$\text{Volume} = 4 \times 4 \times 4 = 64\text{cm}^3$$

The units are cubic for volume.



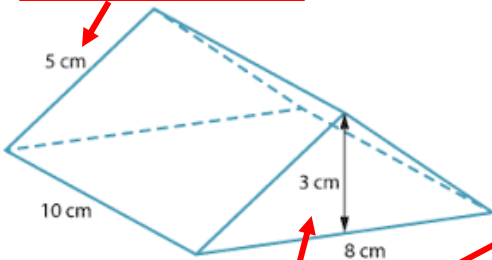
$$\text{Volume} = 10 \times 3 \times 5 = 150\text{cm}^3$$

It doesn't matter which order you multiply in.

Volume - Triangular prism

Prisms: To calculate the volume of a prism we calculate the area of the cross section (end face) and then multiply by the length.

We do not need this.



$$\text{Volume} = \text{Area of the cross section} \times \text{Length}$$

$$\text{Area of triangle} = \frac{3 \times 8}{2} = 12$$

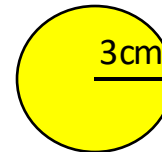
$$\text{Volume} = 12 \times 10 = 120\text{cm}^3$$

Area of the cross section.

Multiply the area of the cross section by 10 which is the length.

Volume - Cylinders

Cylinders: A cylinder is a prism so we use the same formula as for a triangular prism. The difference is that the cross section this time is a circle.



Reminder:

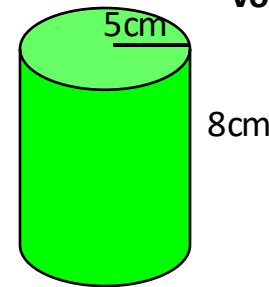
Area of a circle = πr^2 (r is the radius)

$$\text{Area} = \pi \times 3^2 = 9\pi = 28.3\text{cm}^2$$

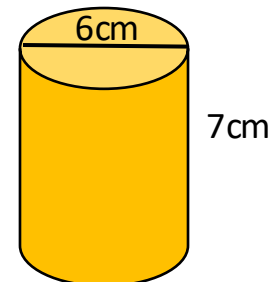
$$\text{Volume} = \text{Area of the cross section} \times \text{Length}$$

$$= \pi r^2 \times \text{Length}$$

Sometimes we leave our answer in terms of π .



$$\begin{aligned} \text{Volume} &= \pi \times 5^2 \times 8 \\ &= 200\pi \\ &= 628.3\text{cm}^3 \end{aligned}$$



We have the diameter so we must divide by 2 to get the radius.

$$\text{Radius} = 6 \div 2 = 3\text{cm}$$

$$\begin{aligned} \text{Volume} &= \pi \times 3^2 \times 7 \\ &= 63\pi \\ &= 197.9\text{cm}^3 \end{aligned}$$

Year Knowledge Organiser PYTHAGORAS'

What do I need to be able to do?

- Explain the definition of a square number and root.
- Calculate the value of square numbers with and without a calculator.
- Round numbers to a suitable degree of accuracy.
- Identify the hypotenuse in a right angled triangle.
- Use Pythagoras' theorem to find the hypotenuse and shorter sides.
- Use Pythagoras' theorem to calculate lengths in 3D solids.

Key Words

Hypotenuse: The longest side in a right angled triangle.

Opposite: The side facing the angle in a right angled triangle.

Adjacent: The side next to the angle given in a right angled triangle.

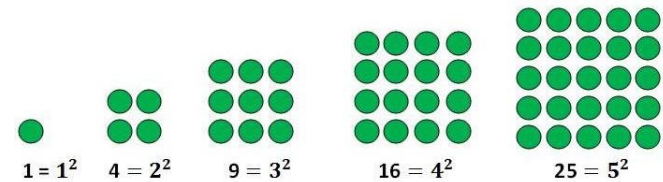
Square number: The result when you multiply a number by itself.

Square root: A square root of a number is a value that, when multiplied by itself, gives the number.

Surd: This is when we leave the value under the square root symbols because it has no exact answer.

Square numbers and roots

Square numbers: This is when we multiply a number by itself, the first 5 square numbers are shown below.



Square roots: This is the number that we started with to get the square numbers.

$$\sqrt{49} = 7 \text{ because } 7 \times 7 \text{ is } 49$$

$$\sqrt{100} = 10 \text{ because } 10 \times 10 \text{ is } 100$$

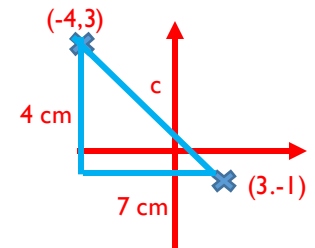
$$\sqrt{120} = 10.95 \text{ to 2dp}$$

Remember the answer is 7 not 7x7.

Finding the distance between two points

Find the distance between (3,-1) and (-4,3).

- 1) Sketch coordinates on an axis.
- 2) Join as a right-angled triangle.
- 3) Find the lengths of the straight sides.
- 4) Use Pythagoras to find the hypotenuse.



$$4^2 + 7^2 = c^2$$

$$65 = c^2$$

$$\sqrt{65} = c \quad c = 8.062257748 \text{ cm}$$

Year 9 Knowledge Organiser PYTHAGORAS'

Calculating the hypotenuse

Always label the hypotenuse first, it is facing the right angle.

$$a^2 + b^2 = c^2$$

1) Substitute your values into the formulae:

$$4^2 + 6^2 = x^2$$

2) Work out the values that you can.

$$16 + 36 = x^2$$

$$52 = x^2$$

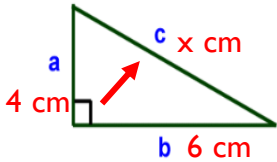
3) Now use inverses to isolate x.

$$52 = x^2$$

$$(\sqrt{}) \quad (\sqrt{})$$

$$\sqrt{52} = x$$

$$7.211102551 \text{ cm} = x$$



This is surd form. Sometimes you will be asked to leave your answer like this.

Calculating shorter sides

1) Substitute your values into the formulae:

$$10^2 + x^2 = 14^2$$

2) Work out the values that you can.

$$100 + x^2 = 196^2$$

3) Now use inverses to isolate x.

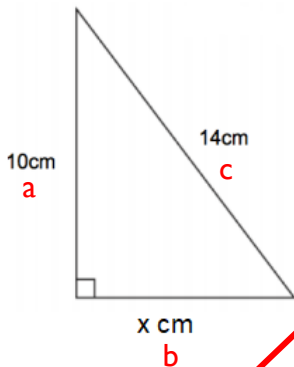
$$100 + x^2 = 196^2$$

$$(-100) \quad (-100)$$

$$x^2 = 96$$

$$(\sqrt{}) \quad (\sqrt{})$$

$$\sqrt{96} = x \quad x = 9.797958971 \text{ cm}$$

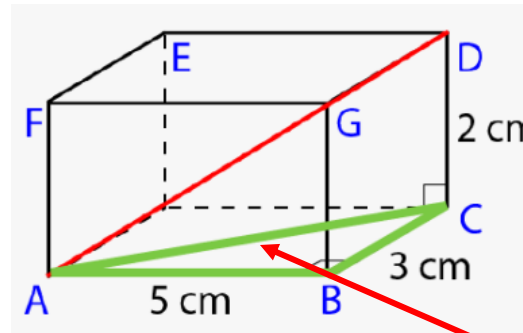


You need to get the numbers on one side, the x on it's own. An extra step is needed.

Calculating in 3D solids

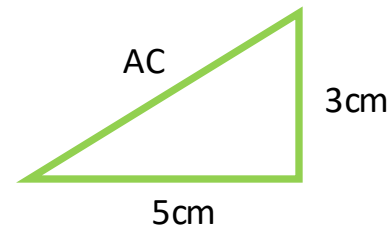
To solve problems in 3D solids we can use Pythagoras' theorem as long as we are working with right angled triangles.

Calculate the length of AD.



We need to use Pythagoras' theorem twice to calculate AD.

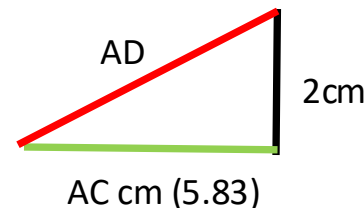
Calculate the length of AC first.



$$AC^2 = 5^2 + 3^2 = 34$$

$$AC = \sqrt{34} = 5.83$$

You then use the value of the diagonal (AC) to calculate AD.



$$AD^2 = 2^2 + AC^2$$

$$AD^2 = 2^2 + 5.83^2 = 37.99$$

$$AD = \sqrt{37.99} = 6.16 \text{ cm}$$



Year 9

Term 3

Knowledge Organiser

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Year 9 Knowledge Organiser LINEAR GRAPHS

What do I need to be able to do?

- Generate y values for a linear graph.
- Draw a linear graph by calculating y values.
- Recognise x= and y= graphs and explain them.
- Calculate the equation of a line.
- Identify parallel and perpendicular equations.
- Calculate the negative reciprocal.

Key Words

Intercept: Where two graphs cross.

Gradient: This describes the steepness of the line.

y-intercept: Where the graph crosses the y-axis.

Linear: A linear graph is a straight line.

Parallel: Where 2 lines never meet. They have the same gradient.

Perpendicular: Two lines that meet at 90° , the gradient is the negative reciprocal.

Substitute: When a letter is replaced by a number.

Reciprocal: This is found by doing 1 divided by the number.

Linear graphs

Linear graphs are straight line graphs. We substitute the x value into the equation to get the y value. Once we have both we can then plot the coordinates and draw the graph.

Draw the graph of $y = 2x - 1$.

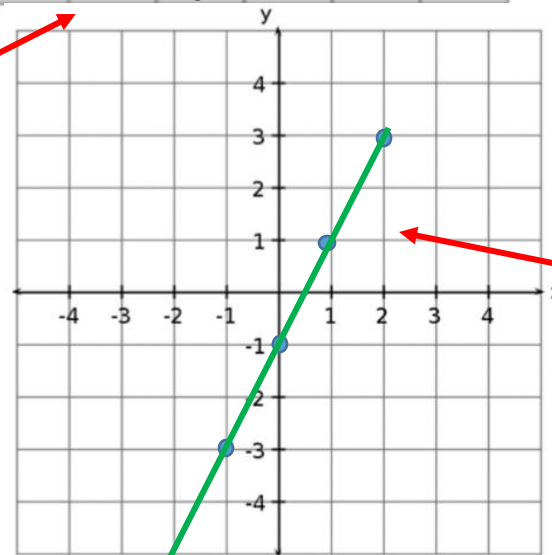
To do this we multiply the x value by 2 and then subtract 1 to get the y value.

$$y = 2x - 1$$

X	-2	-1	0	1	2
Y	-5	-3	-1	1	3

Multiply this value by 2 and then subtract 1 to get the y value.

This coordinate would be $(-2, -5)$.



Don't forget to draw a straight line through all of the coordinates you have plotted.

Notice this graph has a gradient of 2 (the y values go up by 2 each time) and a y-intercept of -1 (the graph cuts through the y axis at -1).

Year 9 Knowledge Organiser LINEAR GRAPHS

Calculating the gradient

Gradient: This is the steepness of the line. The higher the number the steeper the line. We use the formula before to calculate it:

$$\text{Gradient} = \frac{\text{difference in } y}{\text{difference in } x}$$

(3, 4) and (5, 10)

$$\text{Gradient} = \frac{10 - 4}{5 - 3} = \frac{6}{2} = 3$$

Subtract the two x values.

Subtract the two y values.

Gradient = 3

Parallel and perpendicular lines

Parallel lines: The gradient of parallel lines is the same, this is why they never meet.

$$y = 2x + 1$$

$$y = 2x - 4$$

$$y = 2x$$

The gradients are all 2 here so they are all parallel.

Perpendicular lines: The gradient of perpendicular lines is the negative reciprocal, this is why they meet at right angles.

$$y = 2x$$

$$y = -\frac{1}{2}x$$

The negative reciprocal of 2 is $-\frac{1}{2}$.

Equation of a line

Linear equation: The general equation for a linear (straight line) graph is:

$$y = mx + c$$

m = gradient and c = the y intercept

We need to calculate the gradient first and then substitute one of the coordinates into the general equation to calculate the value of c.

Example:

Find the equation of the line going through (3, 4) and (5, 10).

Calculate the gradient and then put it in the place of the m in the general equation.

$$\text{Gradient} = \frac{10 - 4}{5 - 3} = \frac{6}{2} = 3$$

$$y = 3x + c$$

Substitute in (3, 4)

$$4 = 3 \times 3 + c$$

Substitute one of the coordinates into the general equation.
x=3 and y=4.

$$4 = 9 + c$$

Rearrange to find c.

$$-5 = c$$

Put c into the general equation.

$$\text{Equation: } y = 3x - 5$$

Year 9 Knowledge Organiser NON LINEAR GRAPHS

What do I need to be able to do?

- Generate y values for a quadratic graph and draw it.
- Find the turning point of a quadratic.
- Find the roots of a quadratic.
- Generate y values for a cubic graph and draw it.
- Generate y values for a reciprocal graph and draw it.
- Generate y values for an exponential graph and draw it.

Key Words

Intercept: Where two graphs cross.

y-intercept: Where the graph crosses the y-axis.

Turning point: This is the point where the graph changes direction.

Roots: These are the points where the graph cuts through the x axis.

Quadratic: Equations where x^2 is the highest power of x.

Cubic: Equations where x^3 is the highest power of x.

Reciprocal: This is found by doing 1 divided by the number.

Exponential: This is a rapid increase.

Parabola: The shape of a quadratic graph.

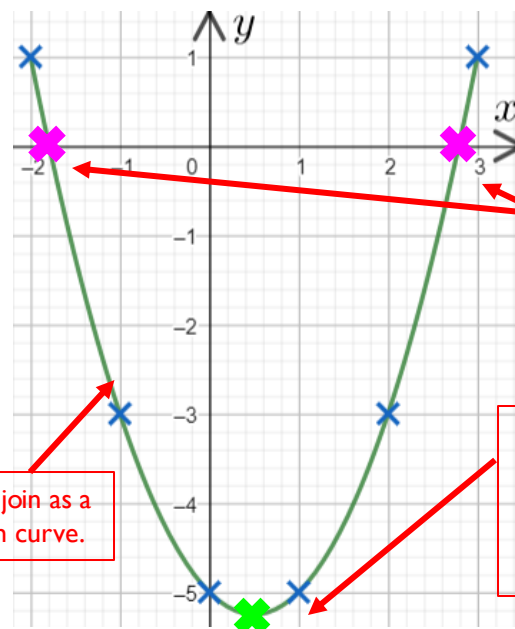
Quadratic graphs

Quadratic graphs: They are graphs of equations where x^2 is their highest power of x. A positive x^2 will give a u shape, a negative x^2 will give an n shape.

$$y = x^2 - x - 5$$

x	-2	-1	0	1	2	3
y	1	-3	-5	-5	-3	1

When $x=3$
 $y = 3^2 - 3 - 5 = 1$.
 We sub in 3 wherever there is an x in the equation.



Roots: This is where the graph cuts through the x axis. Sometimes they are given as values, sometimes as coordinates.

Turning point: This is the coordinate where the graph changes direction. This turning point is a minimum.

Always join as a smooth curve.

Roots = -1.8 and 2.8 or (-1.8,0) and (2.8, 0)

Turning point = (0.5, -5.3)

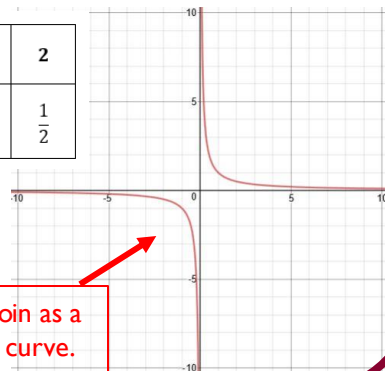
Year 9 Knowledge Organiser NON LINEAR GRAPHS

Reciprocal graphs

Reciprocal graph: Graphs in the form $y = \frac{k}{x}$, where k is a constant.

$$y = \frac{1}{x}$$

x	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2
y	$-\frac{1}{2}$	-1	-2	-4	4	2	1	$\frac{1}{2}$



When $x = -2$, $y = \frac{1}{-2} = -0.5$.
We substitute the x value in as the denominator.

Always join as a smooth curve.

Exponential graphs

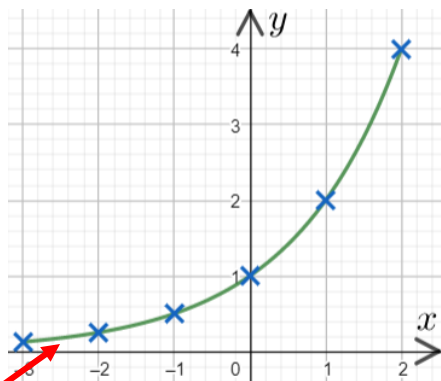
Exponential graphs: Graphs in the form $y = k^x$. They increase rapidly in the y axis and never fall below zero in the x axis.

$$y = 2^x$$

x	-3	-2	-1	0	1	2
y	0.125	0.25	0.5	1	2	4

When $x = -3$, $y = 2^{-3} = 0.125$. We substitute the x value in as the power.

Always join as a smooth curve.



Cubic graphs

Cubic graphs: This type of graphs contains only terms up to and including x^3 . Cubic graphs are curved but can have more than one change of direction.

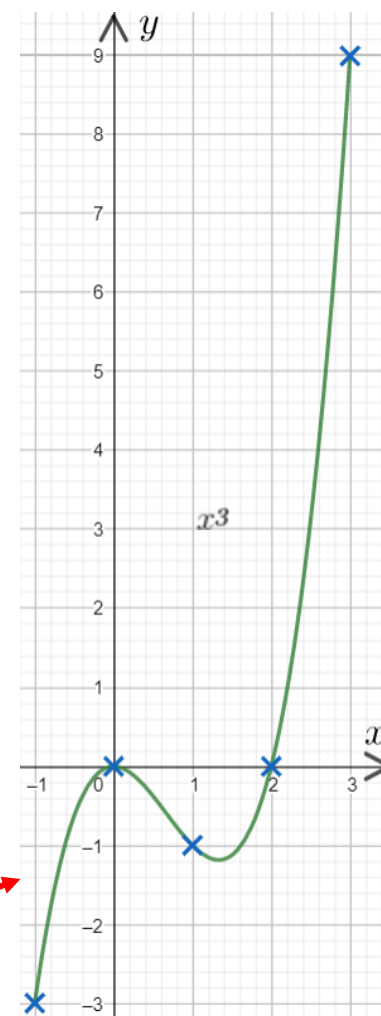
$$y = x^3 - 2x^2$$

x	-1	0	1	2	3
y	-3	0	-1	0	9

We must be very careful when substituting negative values of x in. Remember that $-1^3 = -1$ and $-1^2 = 1$.

When $x = 3$, $y = 3^3 - 2 \times 3^2 = 27 - 18 = 9$.
We substitute the x value in wherever an x appears in the original equation.

Always join as a smooth curve.



Year 9 Knowledge Organiser SIMULTANEOUS EQUATIONS

What do I need to be able to do?

- Solve two step equations.
- Solve simultaneous equations graphically.
- Solve simultaneous equations by elimination.
- Solve simultaneous equations by altering one of the equations.
- Solve simultaneous equations by altering both of the equations.

Key Words

Variable: This refers to a letter representing a number.

Equation: A mathematical statement that shows that two expressions are equal.

Solve: To get the solution or answer to a question.

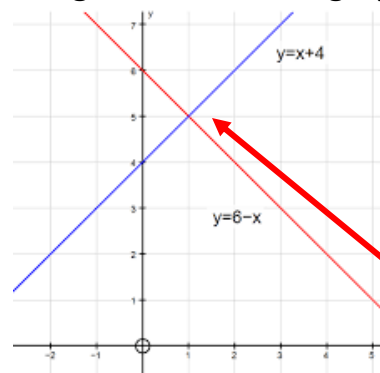
Simultaneous: At the same time.

Coefficient: The number in front of a variable.

Eliminate: Get rid of one of the variables.

Solving graphically

When we solve a pair of simultaneous equations we are actually finding where the two graphs would cross. This is then the solution.



Looking at these two graphs the solution is where they cross each other.

The solution here would be $x=1$ and $y=5$ so $(1,5)$.

This is the solution.

Simultaneous equations – equal variable

To solve a pair of simultaneous equations without a graph we need to make sure that the x values or the y values are equal first.

$$3x + y = 10$$

$$- x + y = 4$$

$$2x = 6$$

$$x = 3$$

$$3x + y = 10$$

$$3 \times 3 + y = 10$$

$$9 + y = 10$$

$$y = 1$$

Solution: $x = 3$ and $y = 1$ or $(3, 1)$

There is only 1 y in both of the equations so we can subtract them straight away.

When we subtract the equations we are just left with the x terms so we can find this value.

Substitute the x value back into one of the original equations to find the value of y .

Always check your values work! $x + y = 4$ so $3 + 1 = 4$.

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Year 9 Knowledge Organiser SIMULTANEOUS EQUATIONS

Simultaneous equations – altering one equation

Here we can see that neither the x or y values have equal coefficients but we can change y into 4y.

$$2x + 4y = 26 \quad 1$$

$$3x - y = 4 \quad 2$$

We can change y into 4y so we only multiply 1 of the equations.

$$1 \quad 2x + 4y = 26$$

$$2 \times 4 \quad 12x - 4y = 16$$

$$+ \quad \underline{\hspace{1cm}}$$

$$14x = 42$$

$$x = 3$$

$$2x + 4y = 26$$

$$2 \times 3 + 4y = 26$$

$$6 + 4y = 26$$

$$4y = 20$$

$$y = 5$$

Once the y values are equal we add the equations to get rid of the y values and solve to find x.

Substitute x into one of the original equations to calculate y.

Solution: $x = 3$ and $y = 5$ or $(3, 5)$

Forming simultaneous equations

Forming: To form the equations we need to use one letter for the DVDs and a different letter for the CDs. Once formed we would then solve as normal.

David buys 2 DVDs and 2 CDs in a shop and in total they cost £18.
Ellie buys 3 DVDs and 2 CDs in the same shop and they cost £22.

Equations: $2x + 2y = 18$

$$3x + 2y = 22$$

where x is the DVDs and y is the CDs.

Both statements are talking about the same 2 items, this is why we can form simultaneous equations.

Simultaneous equations – altering both equations

Looking at this pair of simultaneous equations we can see that neither of the variables are equal and we can't multiply only one of them.

$$2x + 3y = 8 \quad 1$$

$$3x + 2y = 7 \quad 2$$

Neither the x or y values are equal and we cannot make one into the other so we must multiply both.

$$1 \times 3 : 6x + 9y = 24$$

$$2 \times 2 : 6x + 4y = 14$$

Both equations now have the same leading coefficient of 6, we then solve as we would normally.

$$6x + 9y = 24$$

$$- \quad 6x + 4y = 14$$

$$\underline{\hspace{1cm}}$$

$$5y = 10$$

$$y = 2$$

When we subtract the equations we are just left with the y terms so we can find this value.

$$2x + 3y = 8$$

$$2x + 3 \times 2 = 8$$

$$2x + 6 = 8$$

$$2x = 2$$

$$x = 1$$

Substitute y into one of the original equations to calculate x.

Always check your values work!

Solution: $x = 1$ and $y = 2$ or $(1, 2)$

Year 9 Knowledge Organiser BASIC PROBABILITY

What do I need to be able to do?

- Use the correct terminology to describe the probability of a statement.
- Use a probability scale.
- Write the probability of an event as a fraction or decimal.
- Calculate the probability from a table.
- Complete a sample space diagram.
- Complete a probability tree.

Key Words

Probability: The chance of something happening as a numerical value.

Outcome: This is the possible result of the experiment.

Impossible: The outcome cannot happen.

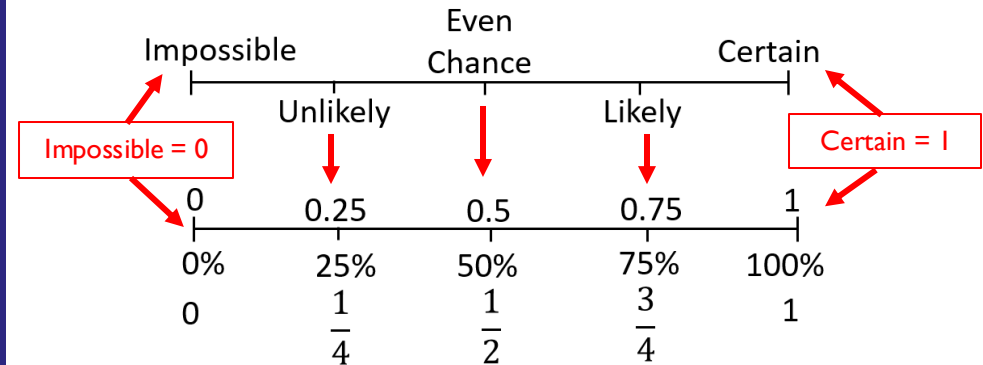
Certain: The outcome will definitely happen.

Even chance: There are two different outcomes each with the same chance of happening.

Expectation: The amount of times you expect an outcome to happen based on probability.

Probability scales

Probability is measured between 0 and 1, it cannot be more than 1. Below are examples of probability scales with the keywords and numerical values.



Probabilities can be written as:

- Fractions
- Decimals
- Percentages

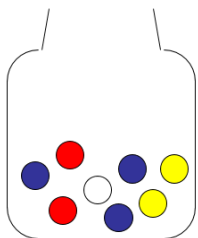
Sample space diagrams

This is a way of recording all the outcomes of 2 events. The sample space opposite records all the possible outcomes when 2 four sided spinners are spun and their totals multiplied together.

×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

This will not always be a multiplication. It could be any operation so make sure you check!

Year 9 Knowledge Organiser BASIC PROBABILITY

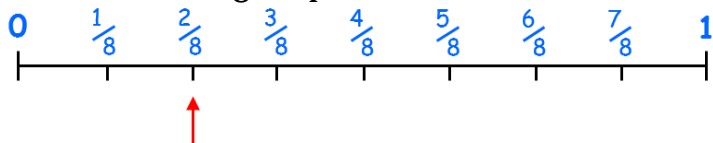


Writing as fractions

Calculate the probability that a bead chosen will be **yellow** and then show your answer on the probability scale.

$$\text{Probability} = \frac{\text{Number of yellow outcomes}}{\text{Total number of outcomes}}$$

$$P(\text{Yellow}) = \frac{2}{8} = \frac{1}{4}$$



Expected outcomes

This is when we have the probability of an event and we predict that if we complete an experiment a given number of times how many times we would expect an event to occur.

If the probability of picking a green counter was $\frac{2}{5}$ and we picked a counter out 40 times then we would expect to get green 16 times.

$$\frac{2}{5} \times 40 = \frac{2}{5} \text{ of } 40 = 16$$

Probability x number of times the experiment is completed.

Probability from a table

To calculate the probability from a table like the one seen below we need to:

1. Add the probabilities we know together.
2. Subtract our total from 1.

Letter	A	B	C	D	E
Probability	0.07	0.15	0.26		0.18

$$0.07 + 0.15 + 0.26 + 0.18 = 0.66$$

$$1 - 0.66 = 0.34$$

Remember that probability must add up to 1.

The probabilities for the different parts of a 4 sided biased spinner are shown in the table below.

Number	1	2	3	4
Probability	0.1	0.2	0.4	

Find the missing probability

The spinner is spun 100 times.

Calculate an estimate for the number of times the spinner will land on number 1

$$0.1 + 0.2 + 0.4 = 0.7$$

$$1 - 0.7 = 0.3$$

Estimate of a 1: $100 \times 0.1 = 10$.

Multiply the probability of the event by the number of times the spinner is spun.

Year 9 Knowledge Organiser MORE PROBABILITY

What do I need to be able to do?

- Complete a two way table and use it to calculate probabilities.
- Read and complete a venn diagram.
- Calculate a probability from a venn diagram.
- Explain the difference between theoretical and experimental probability.
- Complete and use a probability tree.

Key Words

Probability: The chance of something happening as a numerical value.

Two way table: A table of data linked to 2 variables.

Venn diagram: A Venn diagram shows the relationship between a group of different things (a set) in a visual way.

Probability tree: A diagram used to calculate all of the possible outcomes.

Experimental probability: The probability based on an experiment.

Theoretical probability: The probability of what we expect to happen.

Two way tables

Two-way tables are a way of sorting data so that the frequency of each category can be seen quickly and easily.

The two-way table shows some information about the number of students in a school.

	Year Group			Total
	9	10	11	
Boys	149	133	125	407
Girls	154	123	147	424
Total	303	256	272	831

$$407 - (125 + 133) = 149$$

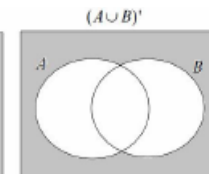
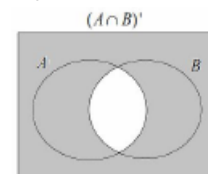
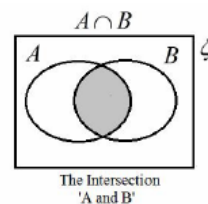
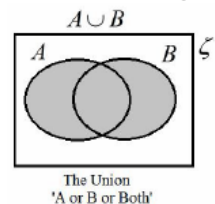
$$256 - 123 = 133$$

$$831 - 407 = 424$$

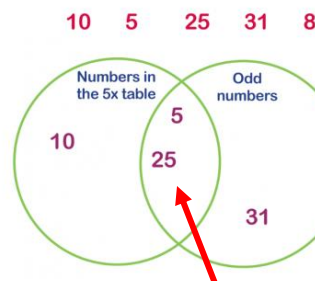
$$P(\text{year 9 girl}) = \frac{154}{831}$$

Venn diagrams

A Venn diagram shows the relationship between a group of different things (a set) in a visual way.



The dash means it is not in this group.



8 is not in A or B.

$A \cap B$ – odd and in the 5 times table.

$$A \cap B = 5, 25$$

$$A \cup B = 5, 10, 25, 31$$

$$(A \cup B)' = 8$$

$$P(A \cap B) = \frac{2}{5}$$

2 numbers in the centre out of the 5 altogether.

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Year 9 Knowledge Organiser MORE PROBABILITY

Theoretical probability

Theoretical: This is the probability that a certain outcome will occur using reasoning or by completing a calculation. It is the expected outcome.

5 appears.

$$P(\text{roll a 5}) = \frac{1}{6}$$



Roll a die once

Theoretical Probability

$$= \frac{\text{Number of favorable (desired) outcomes}}{\text{Total number of possible outcomes}}$$

How many 5's?

How many number in total?

This is an example of theoretical probability because we know that there are 6 numbers on a dice and only one of them is a 5.

Experimental probability

Experimental : This refers to the probability of an event occurring when an experiment was conducted.

5 appears 140 times.

$$P(\text{roll a 5}) = \frac{140}{1000}$$



Roll a die 1000 times.

$$\frac{\text{Number of times the event occurred}}{\text{Number of trials}}$$

How many 5's?

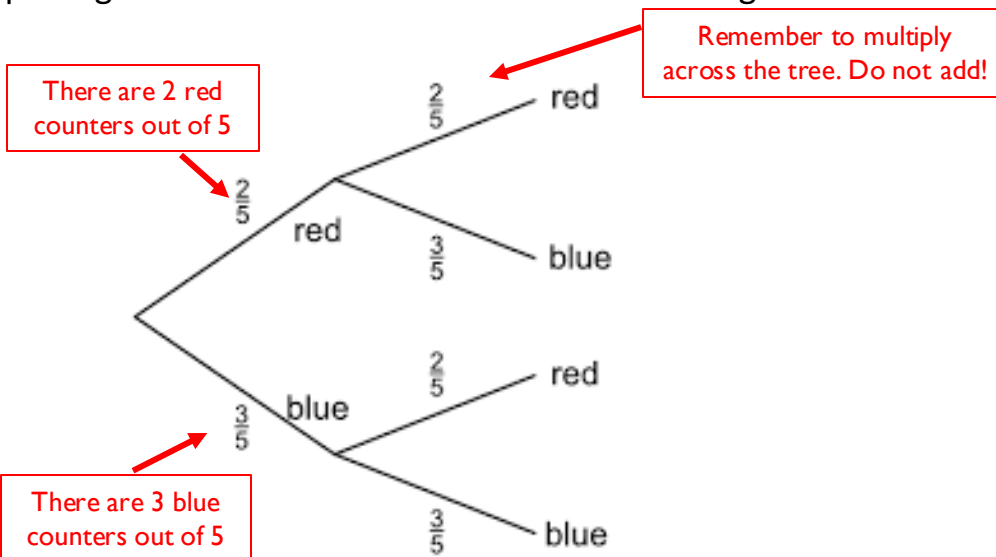
How many times did you do the experiment?

This is an example of experimental probability because we have completed the experiment of rolling the dice. We got 140 5's out of 1000 rolls of the dice.

Probability trees

Probability trees: These are a way of recording the outcomes of multiple events and calculating their probability. To find the probability of an event we multiply across the branches to calculate the probabilities of each outcome.

Here we have a probability tree that shows the probability of picking a red counter or blue counter out of a bag.



$$P(2 \text{ red counters}) = \frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$$

$$P(\text{Not getting 2 reds}) = 1 - \frac{1}{10} = \frac{9}{10}$$

To find the probability of an event not happening you can subtract from 1.