



KS4 Maths Foundation Knowledge Organisers



Knowledge Organiser: 1a Integers and Place Value

What you need to know:

Ordering positive and negative numbers

The further left you go the smaller the number is.

Example – Put the following numbers in ascending order

2 -4 5 -2 3 -1

Ascending – smallest to biggest = -4, -2, -1, 2, 3, 5

Calculating with positive numbers

When you are unable to complete a calculation mentally use a written method. The most common method for addition, subtraction and multiplication is column method; for division use Bus Stop method.

Examples

$$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \\ \hline \end{array}$$

$$\begin{array}{r} 8121 \\ 932 \\ - 457 \\ \hline 475 \end{array}$$

$$\begin{array}{r} 24 \\ \times 16 \\ \hline 240 \\ 144 \\ \hline 384 \end{array}$$

$$5 \overline{) 158} \begin{array}{r} 31 \\ 79.40 \end{array}$$

When adding or subtraction start with the units column.

Powers of 10

Use a place value grid to help if needed. If there are any gaps before the decimal point, you need to fill them in with zeros.

Remember:

To multiply by 10, move the digits on a place value grid left by one column.
To multiply by 100, move two columns; to multiply by 1000, 3 columns etc.
To divide by 10, move the digits on a place value grid right by one column.
To divide by 100, move two columns, divide by 1000, 3 columns etc.

Examples:

$$53 \times 10 = 530$$

$$18 \times 100 = 1800$$

$$60 \times 10^3 = 60000$$

$$53 \div 10 = 5.3$$

$$18 \div 100 = 0.18$$

$$60 \div 10^3 = 0.060$$

$$10^3 = 1000$$



Key Terms:

Ascending – smallest to biggest

Descending – biggest to smallest

Approximate - to estimate a number, amount or total often using rounding to make them easier to calculate with

Integer – a WHOLE number that is positive or negative

Negative – a number less than 0 with a minus sign

Place Holder – we use 0 as a place holder

Place Value – The value of a digit depending on its position in the number

You need to be able to:

- Use and order positive and negative numbers
- Use the symbols $<$, $>$ and understand the $\neq$ symbol
- Add, subtract, multiply and divide positive and negative numbers
- Multiply and divide by powers of 10
- Round numbers to a given power of 10
- Understand the order of operations

Reminder:

Inequality signs

$<$ less than
 $>$ greater than
 $=$ equal to
 $\neq$ not equal to



Knowledge Organiser: 1a Integers and Place Value

What you need to know:

Directed Numbers – positive and negative numbers

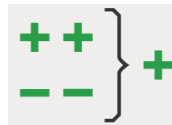
Adding and Subtracting

Remember:

Subtract when two different signs appear **next to each other**

Add when two of the same signs appear **next to each other**

You can draw and use a number line to help you with adding and subtracting



Examples:

$$3 - 7 = -4$$

$$-2 - 9 = -11$$

$$-5 + 2 = -3$$

$$5 - -2 = 7$$

$$-4 + -5 = -9$$

$$-8 - -2 = -6$$

$$5 + 2 = 7$$

$$-4 - 5 = -9$$

$$-8 + 2 = -6$$

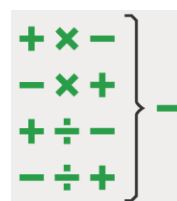
In the last three examples the two signs appear next to each other

Multiplying and Dividing

Remember:

When the signs are **different** the answer is **negative**

When the signs are the **same** the answer is **positive**



Examples:

$$5 \times -4 = -20$$

$$-3 \times -8 = 24$$

$$-6 \times 2 = -12$$

$$45 \div -5 = -9$$

$$-100 \div -10 = 10$$

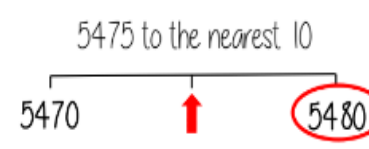
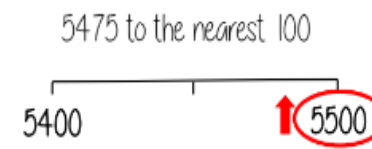
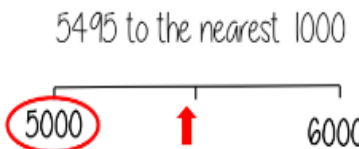
$$-18 \div 9 = -2$$

Tip

Always check your answers by using rounding and inverse operations

Example: $9 \times 20 = 180$
Check $180 \div 20 = 9$

Rounding to the nearest power of ten



To round to the nearest 10, 100 or 1000 look at the digit in the corresponding column.

Look at the next digit.

5 or more 'round up' (increase by 1)

4 or less 'round down' (keep the same)

Fill any spaces with zeros.

Round 4,853 to the nearest 10, 100 and 1,000.

Round 76,982 to the nearest 10, 100 and 1,000.

485|3 to the nearest 10 is 4,850

48|53 to the nearest 100 is 4,900

4|853 to the nearest 1,000 is 5,000

7698|2 to the nearest 10 is 76,980

769|82 to the nearest 100 is 77,000

76|982 to the nearest 1,000 is 77,000

Order of Operations - BIDMAS

1st

B rackets

2nd

I ndices

3rd

D ivision

4th

M ultiplication

A ddition

S ubtraction

This is the order in which you should work out your calculations. Any brackets work them out first. Then any indices. Then division and multiplication (in the order they appear). Finally any addition and subtraction (in the order they appear)

Examples:

$$4 + 2 \times 3$$

$$\downarrow$$

$$4 + 6 = 10$$

$$3 + (7 \times 2 - 5)$$

$$\downarrow$$

$$3 + 9 = 12$$

$$(8 - 2) \times 3^2$$

$$\downarrow$$

$$6 \times 3^2$$

$$\downarrow$$

$$6 \times 9 = 54$$



Knowledge Organiser: 1b Decimals

What you need to know:

Column addition and subtraction

$$\begin{array}{r} 5.649 \\ + 39.27 \\ \hline 44.919 \end{array}$$

Addition: Starting with the digit on the right, add each column in turn. Regroup tenths, tens, hundreds etc as required.

$$\begin{array}{r} 3.47 \\ - 1.59 \\ \hline 1.88 \end{array}$$

You must remember to borrow if you can't subtract with the numbers you have.

Subtraction: Starting with the digit on the right, subtract each column in turn. Exchange tenths, tens, hundreds etc as required.

Column multiplication

Question: 1.54×2.6

Now 154×26

1	5	4	
×	2	6	
	9	2	4
3	0	8	0
4	0	0	4
1	1		

We multiplied by 1000 to get rid of the decimal points because there are 3 numbers in total after the point in the question.

Multiply the whole numbers.

Divide your answer by 1000.

Multiplication: Remove any decimal points from your values before multiplying. Then multiply as you would normally. When you have finished multiplying and have added to get your total remember to then divide by the multiple of 10 that you multiplied by to remove the decimal point at the beginning.

Answer = 4.004

Key Terms:

Add: Finding the total of 2 or more number.

Subtract: Finding the difference between 2 numbers.

Multiply: Increasing a value by a given times table.

Divide: Share a given number using a specific times table.

Decimal: A number that has digits that are smaller than one whole. It has a decimal point.

Round: Changing the given number to a value that it is close to.

Estimate: Calculate an approximate answer to a calculation by rounding the values.

Compare: Examine the difference between the quantities.

You need to be able to:

- Add and subtract decimals using the column method.
- Multiply decimals by whole numbers and decimals by decimals using a written method.
- Divide decimals using the bus stop method.
- Use rounding to estimate values to calculations.
- Order decimals from smallest to biggest.
- Round values to the nearest integer.
- Round answers to a given number of decimal places or significant figures.



Knowledge Organiser: 1b Decimals

What you need to know:

Division: Set up your question as shown. Starting from the left divide the number under the bus stop by the number on the outside. Any remainders must be carried to the next value along. Continue this process until you have got to the end of the number you are dividing.

$$8.12 \div 4$$

	2	.	0	3
4	8	.	1	2

You must carry any remainders here.

To divide a decimal by a decimal we need to multiply both values until the number we are dividing by is a whole number. Then we would divide as before.

$$\begin{array}{r} 0.5 \overline{) 6.85} \\ 5 \overline{) 68.5} \end{array} \rightarrow$$

don't need

Rounding

To make a number simpler but keep its value close to what it was.

If the digit to the right of the rounding digit is less than 5, round down. If the digit to the right of the rounding digit is 5 or more, round up.

For example:

7.44 rounded to 1 decimal place is 7.4, because 7.44 is closer to 7.4 than 7.5.

0.38 rounded to 1 decimal place is 0.4 because 0.38 is closer to 0.4 than 0.3.

When rounding to significant figures we need to look at the first non-zero value and use this to round appropriately.

7.44 rounded to 1 significant figure is 7 because 7.44 is closer to 7 than 8.

0.567 rounded to 2 significant figures is 0.57 because 0.567 is closer to 0.57 than 0.56.

Always check whether the question wants you to round to a given number of decimal places or significant figures because they are different.

Comparing decimals: It is important when comparing decimals to compare each digit which is in the same place value. For example, compare the tenths with each other because they are in the same place.



Compare the tenths – 3 is less than 6 so we know that 2.37 must be smaller than 2.62.

Ordering decimals: When ordering decimals it is important to ensure that all the decimals have the same number of digits.

For example:

0.3, 0.43, 0.03, 0.043 would become 0.300, 0.430, 0.030, 0.043

It is then easy to order them: 0.03, 0.043, 0.3, 0.43

You must only use the additional zeros to help you order the decimals, they must be written as they were in the question as your answer.

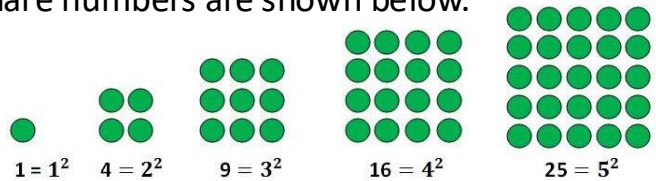


Knowledge Organiser : 1c Indices, Powers and Roots

What you need to know:

Squares, cubes and roots

Square numbers: This is when we multiply a number by itself, the first 5 square numbers are shown below.



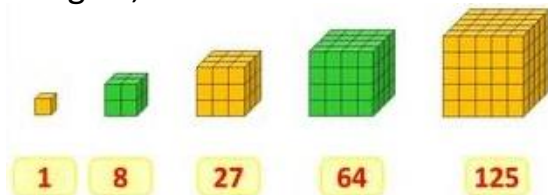
Square roots: This is the number that we started with to get the square numbers.

$$\sqrt{49} = 7 \text{ because } 7 \times 7 \text{ is } 49$$

Remember the answer is 7 not 7×7 .

$$\sqrt{100} = 10 \text{ because } 10 \times 10 \text{ is } 100$$

Cube numbers: This is when we multiply a number by itself and then by itself again, the first 5 cube numbers are shown below.



Index form

Index number: An index number is a number which is raised to a power. The power, also known as the index, tells you how many times you have to multiply the number by itself.

2^5 is the index notation.

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

Key Terms:

Square: A square number is the result of multiplying a number by itself.

Cube: A cube number is the result of multiplying a number by itself twice.

Root: A root is the reverse of a power.

Indices: These are the squares, cubes and powers.

Operation: In maths these are the functions $\times \div + -$.

You need to be able to:

- Recognise and calculate square numbers and roots.
- Recognise and calculate cube numbers and roots.
- Complete calculations with a mixture of powers and roots.
- Use BIDMAS to complete calculations.
- Use the laws of indices to simplify expressions.



Knowledge Organiser: 1c Indices, Powers and Roots

What you need to know:

Laws of indices

Multiplication law: When multiplying with the same base (number/letter) we add the powers.

$$\text{General rule: } a^m \times a^n = a^{m+n}$$

$$2^5 \times 2^7 = 2^{5+7} = 2^{12}$$

$$x^3 \times x^8 = x^{3+8} = x^{11}$$

When multiplying the terms we add the powers together.

Division law: When dividing with the same base (number/letter) we subtract the powers.

$$\text{General rule: } a^m \div a^n = a^{m-n}$$

$$2^{14} \div 2^7 = 2^{14-7} = 2^7$$

$$x^{10} \div x^8 = x^{10-8} = x^2$$

When dividing the terms we subtract the powers together.

Brackets law: When raising a power to another power we multiply the powers together.

$$\text{General rule: } (a^m)^n = a^{m \times n}$$

$$(5^4)^2 = 5^{4 \times 2} = 5^8$$

$$(h^9)^3 = h^{9 \times 3} = h^{27}$$

When raising to a power we multiply the powers together.

BIDMAS – order of operations

B Brackets

I Indices

D Division

M Multiplication

A Addition

S Subtraction

If a calculation only contains the circled operations, then you need to work from left to right.

$$\begin{array}{ccccccc} 5 & \times & 4 & - & 8 & \div & 2 \\ \underbrace{} & & \underbrace{} & & & & \\ 20 & - & 4 & = & 16 & & \end{array}$$

This question can be split into two separate calculations which are then combined to get the answer.

We need to deal with the powers inside the brackets first by calculating 2^2 .

Once the bracket has been fully calculated we then look at the operations on the outside of the bracket.

$$(2^2 + 6)^2 \times 4 - 8$$

$$(4 + 6)^2 \times 4 - 8$$

$$(10)^2 \times 4 - 8$$

$$100 \times 4 - 8$$

$$400 - 8 = 392$$



Knowledge Organiser: 1d Factors, Multiples and Primes

What you need to know:

Multiples and factors

Multiples: The result of multiplying a number by an integer. It is the times table of a number.

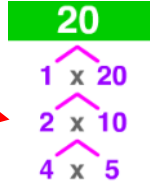
Multiples of 4: 4, 8, 12, 16, 20 ...

Multiples of 5: 5, 10, 15, 20, 25...

Multiples are the
list of times tables.

Factors: A number that divides exactly into another number without a remainder. It is often helpful to write them in pairs.

Write them in
pairs first so you
don't miss any!



Factors of 20 = 1, 2, 4, 5, 10, 20

Prime numbers

Prime: This is a number that has exactly 2 distinct factors; 1 and itself.

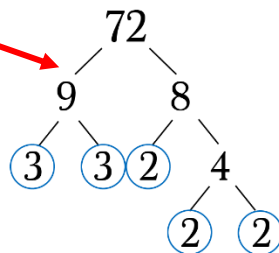
2 is the only even prime. The first 10 prime numbers are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

These are not the
only prime
numbers.

Prime factorisation: This is when we split a number into its prime factors using a factor tree. We circle the prime factors.

We need to find
pairs of numbers
that multiply to
give the number
above.



If a number is
repeated we write
it as a power.

$$72 = 2^3 \times 3^2$$

Key Terms:

Prime number: A prime is a number that has exactly two distinct factors which are 1 and itself.

Multiple: A number in the given number's times table.

Factor: A number that divides into another number exactly.

LCM: The smallest number that is in the times tables of the given numbers.

HCF: The biggest number that divides exactly into two or more numbers.

You need to be able to:

- Identify factors and multiples.
- Identify a prime number.
- Complete a prime factor tree and write the number in index notation.
- Calculate HCF and LCM of two values using an appropriate method.

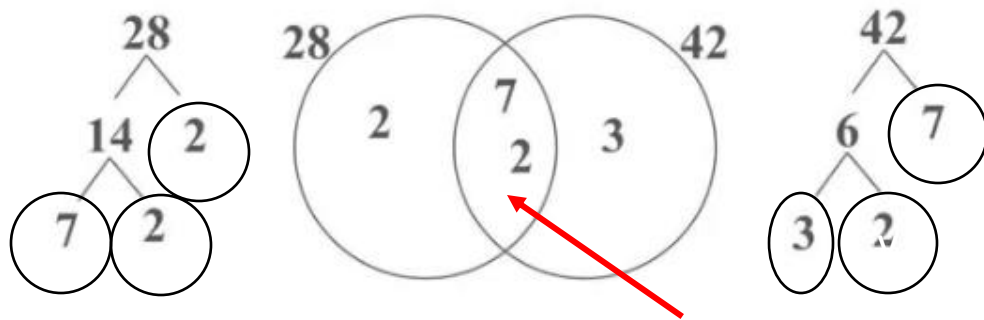


Knowledge Organiser: 1d Factors, Multiples and Primes

What you need to know:

HCF

This is where we find the biggest number that divides exactly into two or more numbers.



The prime numbers (the circled numbers go in the Venn diagram.

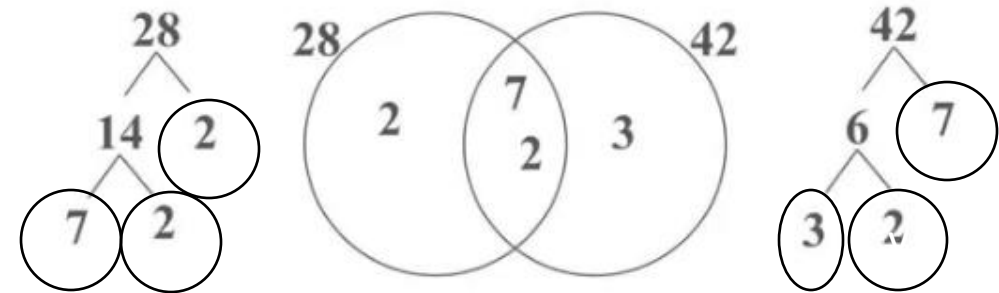
To calculate the HCF we multiply all of the numbers in the intersection.

The HCF of 28 and 42 = $7 \times 2 = 14$

Remember numbers that appear in both prime factor tree's go in the middle of the Venn diagram.

LCM

This is where we find the smallest number that appears in the given numbers times tables.



There are 2 different ways of calculating the LCM:

Method 1:

Multiply all of the numbers in the Venn diagram = $2 \times 7 \times 2 \times 3 = 84$

Method 2:

Multiply the large number outside the Venn diagram by the small numbers in the opposite circle:

$28 \times 3 = 84$ or $42 \times 2 = 84$

All 3 calculations give you the same answer so you could do all 3 to check your answer is correct.

The LCM of 28 and 42 = 84



Knowledge Organiser: 2a Algebra - The Basics

What you need to know:

Algebraic Notation

In algebra, we use specific notation for different calculations

We group letters together

$a + a + a$
means
3 lots of a
 $3 \times a$

$b + b$
means
2 lots of b
 $2 \times b$

We use indices/powers

$a \times a$
 $= a^2$
(a squared)

$b \times b \times b$
 $= b^3$
(b cubed)

We do not use multiplication signs

$3 \times a = 3a$

$a \times b = ab$
 $a \times b \times c = abc$

We write division using fraction notation

$a \div 2$
is written as
 $\frac{a}{2}$ or $\frac{1}{2}a$

$b \div 3$
is written as
 $\frac{b}{3}$ or $\frac{1}{3}b$

Expressions, Equations, Formula and Identities

Here are some examples of each:

Expression	Equation	Formula	Identity
$a+b$	$4x = 12$	$A = bh$	$2 + 3 \equiv 3 + 2$
xy	$\frac{3-x}{2} = 7$	$V = 4/3\pi r^3$	$5(x+4) \equiv 5x + 20$
$\frac{5m}{2n}$	$7y - 5 = -2$	$S = D/T$	$x \times b \equiv ab$

We use three lines to show identities $\equiv$

Collecting Like Terms

Simplify the expression: $4w + 3 + 2w - 1$

$4w + 3 + 2w - 1$ (Now Group Like Terms)

$= 4w + 2w + 3 - 1$ (Combine Like Terms)

$= 6w + 2$

$= 6w + 2$ ✓

$$4x^2 + 3xy - 14x + 7xy + x^2$$

$$5x^2 + 10xy - 14x$$

Note – you can only collect terms that have the same power eg $5x + 4x^2 \neq 9x^2$

Key Terms:

Equation – an equation is a statement with an equals sign, stating that two expressions are equal in value

Identity – an equation that is always true, no matter what values are substituted

Formula – expresses the relationship between two or more unknown values

Expression – a mathematical sentence that does not have an equals sign

Collect 'Like' terms: Like terms are terms whose variables are the same (4y and 8y) To collect like terms means to add or subtract them

You need to be able to:

- Use algebraic notation and symbols correctly
- Write an expression
- Identify an expression, equation, formula and identity from a list
- Simplify expressions by collecting like terms
- Simplify expressions by multiplying and dividing
- Know and use index laws

Reminder:

In algebra, letters are used to stand for values that can change (**variables**) or for values that are not known (unknowns).

A **term** is a number or letter on its own, or numbers and letters multiplied together



Knowledge Organiser: 2a Algebra - The Basics

What you need to know:

Writing Expressions

Phrases or verbal expressions

Numerical or algebraic expressions

Three times	Some number	$3 \times a$
Ten plus	a number	$10 + b$
One subtracted from six		$6 - 1$
Four divided by	some number	$4 \div y$
Eight less than	an unknown number	$x - 8$
Two multiplied by	twelve	2×12
Seven more than	twice a number	$2c + 7$

Worded Example

Pens are sold in packs of 6 and rulers are sold in boxes of 10. A teacher buys p packs of pens and r boxes of rulers. Write an expression for the total number of pens and rulers bought.

6 pens in each pack, so the number of pens bought is $6 \times p = 6p$

10 rulers in each box, so the number of rulers bought is $10 \times r = 10r$

The number of pens and rulers bought is $6p + 10r$

Simplifying Expressions

Simplify the following:

$$a) y \times y = y^2$$

$$b) c \times c \times c = y^3$$

$$c) 5a \times 2b = 10ab$$

Tip – remember $2y$ is not the same as y^2

Simplify the following:

$$a) 10e \div 2 = 5e$$

$$b) \frac{20b^2}{5} = 4b$$

$$c) \frac{18xy^2}{6xy} = 3y$$

You could also use the identity sign ($\equiv$) when simplifying expressions

Index Laws

Simplify the following:

$$a^3 \times a^4$$

If we start by writing it out in full:

$$a^3 = a \times a \times a$$

$$a^4 = a \times a \times a \times a$$

$$\therefore a \times a \times a \times a \times a \times a \times a = a^7$$

To multiply together two identical values or variables (letters) that are presented in index form, add the powers.

$$d^8 \times d^2 = d^{10} \quad e^{-3} \times e^5 = e^2 \quad f \times f^3 = f^4$$

Simplify the following:

$$\frac{m^5}{m^3}$$

If we start by writing it out in full:

$$m^5 = m \times m \times m \times m \times m$$

$$m^3 = m \times m \times m \times m \times m$$

$$\therefore \frac{m \times m \times m \times m \times m}{m \times m \times m} = m^2$$

To divide two identical values or variables (letters) that are presented in index form, subtract the powers

$$p^6 \div p^3 = p^3 \quad \frac{s^3}{s^7} = s^{-4}$$

Simplify the following:

$$(y^4)^2$$

If we start by writing it out in full:

$$y^4 \times y^4 = y^8$$

To raise a value or variable (letter) presented in index form to another index, multiply the powers together

$$(m^7)^3 = m^{21} \quad (n^6)^{-4} = n^{-24}$$

Reminder:

1.	$a^n \times a^m = a^{n+m}$
2.	$a^n \div a^m = a^{n-m}$
3.	$(a^n)^m = a^{n \times m}$
4.	$a^1 = a$
5.	$a^0 = 1$
6.	$(ab)^n = a^n b^n$
7.	$a^{-n} = \frac{1}{a^n}$



Knowledge Organiser: 2a Expressions and Substitution

What you need to know:

Expressions

Expression: An expression is a group of numbers, letters and operation symbols. It is important that you read the words carefully and work out their meaning before deciding on the appropriate operation. Expressions never have an equals sign.

More than = add
Less than = subtract
Always check the wording carefully.

Add 14 to a	$a + 14$
Subtract 20 from b	$b - 20$
Multiply c by 4	$4c$
12 more than d	$d + 12$
Multiply e by 3 and subtract 5	$3e - 5$
Add 12 to f and then multiply by 2	$2(f + 12)$

Equations

Equation: An equation is a number statement with an equals sign. Expressions on either side of the equals sign are of equal value. There is always a solution or answer to an equation.

To solve the equation you would then do the inverse of the operation that you can see. Inverse means the opposite.

$$\begin{aligned}d + 12 &= 30 \\d &= 30 - 12 \\d &= 18\end{aligned}$$

$$\begin{aligned}a + 14 &= 20 \\b - 20 &= 15 \\4c &= 28 \\d + 12 &= 30 \\3e - 5 &= 10 \\2(f + 12) &= 44\end{aligned}$$

Key Terms:

Formula: A rule written using symbols that describe a relationship between different quantities.

Expression: Shows a mathematical relationship whereby there is no solution.

Equation: A mathematical statement that shows that two expressions are equal.

Term: This is a number and a letter put together, for example $3x$ is a term.

You need to be able to:

- Simplify expressions by collecting like terms.
- Write an expression from a given sentence.
- Substitute into an expression or formula.

REMEMBER:

$$a + a = 2a$$

$$a \times a = a^2$$

$$3 \times a = 3a$$

$$a^2 + a^2 = 2a^2$$



Knowledge Organiser: 2b Expressions and Substitution

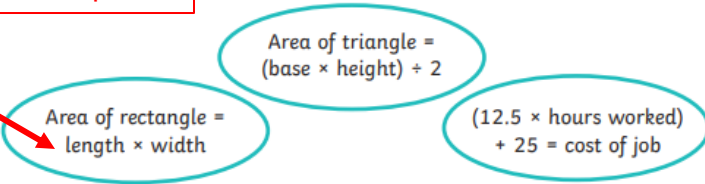
What you need to know:

Formulae

Formulae: The word formula has two possible plural forms, formulae and formulas.

A formula is a special type of equation that shows the relationship between different variables. Formulae are often used in geometry topics to find area and volume.

Values would be substituted into the formula in the correct place.



Substitution - Formula

Substitution: This is where we replace words or letters for the number that they are worth.

For example: The time in minutes to cook a chicken is given by the formula:

Time = 40 minutes per kilogram plus 20 minutes

- a) 5kg chicken = $40 \times 5 + 20 = 220$ minutes
- b) 2.5kg chicken = $40 \times 2.5 = 120$ minutes

Simplifying expressions

The operation symbol in front of the term tells you what to do.

Simplify:

$$4a + 3b - a + 2b = 3a + 5b$$

Collecting like terms: You can only collect terms with the same letter together. The 'a' terms can only be collected with other 'a' terms. The operation symbol in front of the terms tells you what to do with it.

Expand and simplify:

$$2(4a + 2b) - 2(a + 3b)$$

$$8a + 4b - 2a - 6b = 6a - 2b$$

Expanding and collecting like terms: You need to first expand the brackets separately and then collect the terms with the same letter as you did in the previous question.

Substitution - Expression

Substitution: This is where we replace the letter we see for the number that it is worth.

For example: If $w = 6$ and $y = 5$

Remember that $3y$ means 3 multiplied by the value of y .

a) $w + 5 = 6 + 5 = 11$

b) $3y - 2 = 3 \times 5 - 2 = 15 - 2 = 13$

c) $8w + 2y = 8 \times 6 + 2 \times 5 = 48 + 10 = 58$



Knowledge Organiser: 3a Tables, Charts and Graphs

What you need to know:

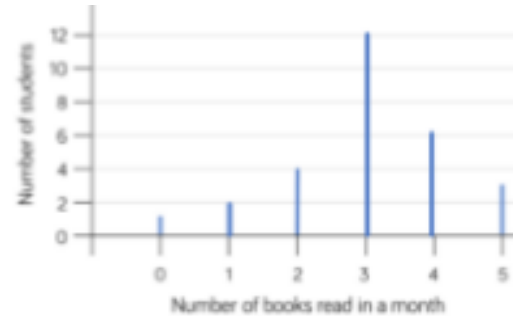
Frequency Table

Tally marks are used to help count things. Each vertical line represents one unit. The fifth tally mark goes down across the first four to make it easier to count. The frequency column is completed after all the data has been collected.

You must represent 5 like this.

Eye Colour	Tally	Frequency
brown		6
blue		8
green		3
grey		4
hazel		5

Vertical Line Chart



- Gaps between the lines.
- Clearly labelled axes.
- Scale for the axes.
- Discrete data only.

Key Terms

Mean: Average of all numbers (add all and divide by how many).

Median: The middle of an ordered list of numbers.

Mode: The value that occurs the most times.

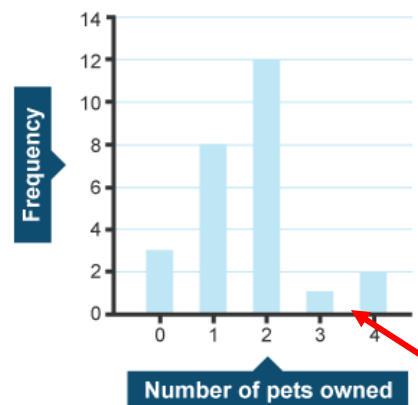
Range: The difference between the lowest and highest values.

Frequency: The number which tells us how many pieces of data there are.

Discrete Data: Data that can only be set values e.g. you cannot have half of a person so counting people would be discrete data.

Continuous Data: Data that can be any value e.g. height and time.

Bar Charts

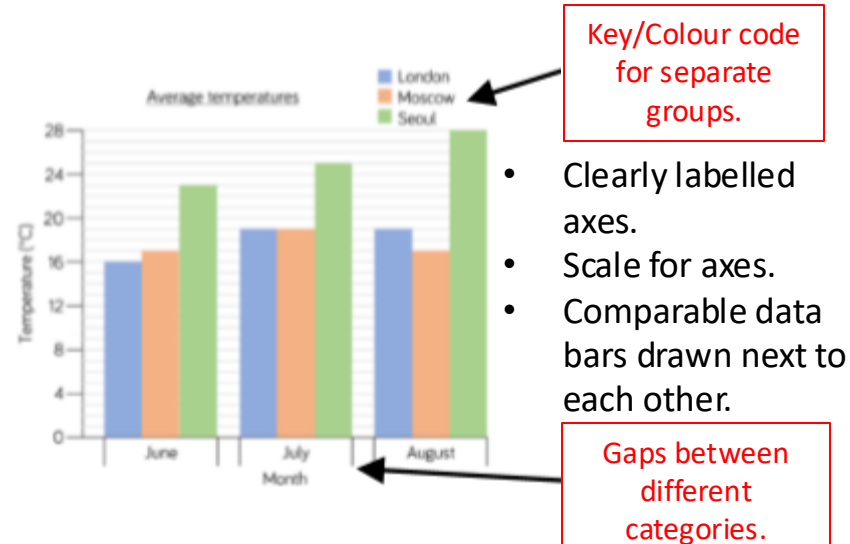


A bar chart has a horizontal axis and a vertical axis. The x axis is for the type of data and the y axis shows the frequency. The bars show the data value of each category. There must be a gap between each bar and the scale must increase in the same sized intervals and the axes must be labelled.

You must include gaps and labels.

Dual Bar Charts

Compares multiple groups of data.



Key/Colour code for separate groups.

- Clearly labelled axes.
- Scale for axes.
- Comparable data bars drawn next to each other.

Gaps between different categories.



Knowledge Organiser: 3a Tables, Charts and Graphs

What you need to know:

Pictograms

Pictograms are similar to bar charts, but the data is shown in pictures. A pictogram **must have a key** so that you know what a full image represents. Looking at this diagram:

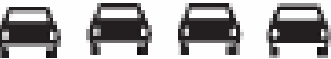
Black = $4 + 4 + 2 = 10$ cars

Red = $4 + 4 + 4 = 12$ cars

Green = 2 cars

Others = $4 + 4 + 4 + 4 = 16$ cars

This represents 2 cars because it is half of the diagram in the key.

Black 
Red 
Green 
Others 

Key

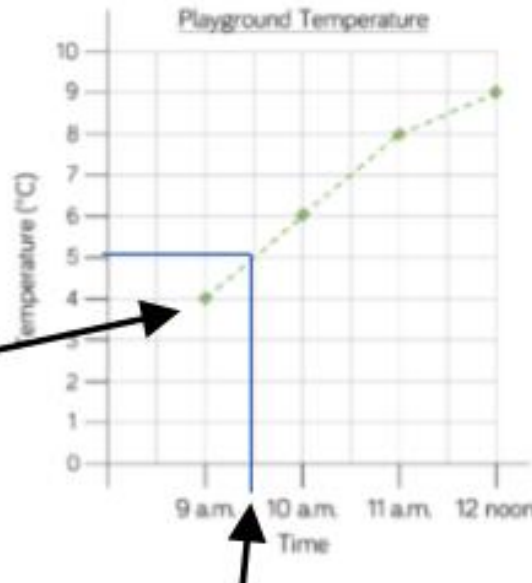
 = 4 cars

Line Graphs/Time Series

- Commonly used to show changes over time.
- The points are the recorded information and the lines join the points.

Line graphs do NOT need to start from 0.

More than one piece of data can be plotted on the same graph to compare data.



It is possible to make estimates from the line e.g. temperature at 9.30am is 5 degrees Celsius.

Stem and leaf diagrams

Here is a list of numbers and the stem and leaf diagram:

68, 75, 77, 79, 80, 82, 92, 96, 96, 97

Stem	Leaf
6	8
7	5 7 9
8	0 2
9	2 6 6 7

The 'leaves' must be from smallest to biggest in each row.

Key 6|8 = 68

You must include a key to explain what the stem and leaf shows.

Mode = 96 because 96 appears twice.

Median = 81 because 81 is in the middle of 80 and 82.

Range = $97 - 68 = 29$.

Mean = $\frac{68+75+77+79+80+82+92+96+96+97}{10} = 84.2$

We calculate these in the same way we would from a list.

Two Way Tables

A table that organises data around two categories.

Fill out the information step by step using the information given:

Question: Complete the 2 way table below.

	Left Handed	Right Handed	Total
Boys	10		58
Girls			42
Total		84	100

Both need to add to make 100 so the missing number is 16.

Answer: Step 1, fill out the easy parts (the totals)

	Left Handed	Right Handed	Total
Boys	10	48	58
Girls			42
Total	16	84	100

Both need to add to make 100 so the missing number is 42.

Answer: Step 2, fill out the remaining parts

	Left Handed	Right Handed	Total
Boys	10	48	58
Girls	6	36	42
Total	16	84	100

Both need to add to make 16 so the missing number is 6.

Both need to add to make 42 so the missing number is 36.



Knowledge Organiser: 3b Pie Charts

What you need to know:

Pie charts

Pie charts represent **discrete data**. A circle is divided into segments, where each segment represents a data category. The size of each segment matches its proportion of the total amount.

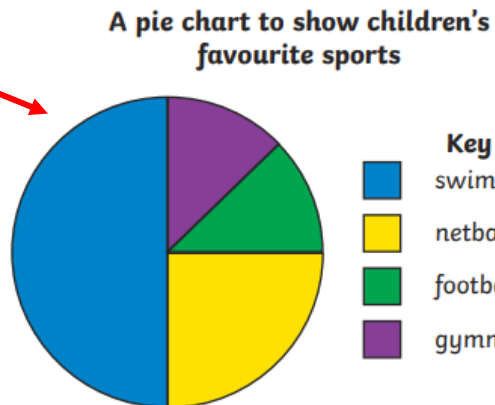
Sport	Frequency	Angle
Swimming	12	$12 \times 15 = 180^\circ$
Netball	6	$6 \times 15 = 90^\circ$
Football	3	$3 \times 15 = 45^\circ$
Gymnastics	3	$3 \times 15 = 45^\circ$

Total = 24

Each person: $360^\circ \div 24 = 15^\circ$

1. Find the total frequency.
2. Calculate one person by : $360^\circ \div \text{total frequency}$.
3. Multiply each frequency by this value to get the angle size for each section.

You must use a protractor carefully to measure each section.



Key

- swimming
- netball
- football
- gymnastics

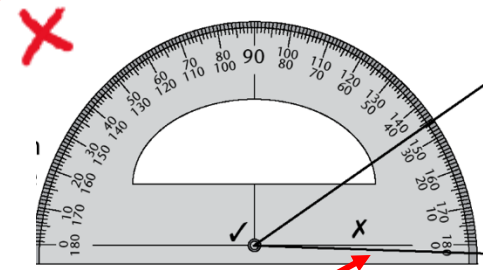
You must label each section or use a key.

Key Terms:

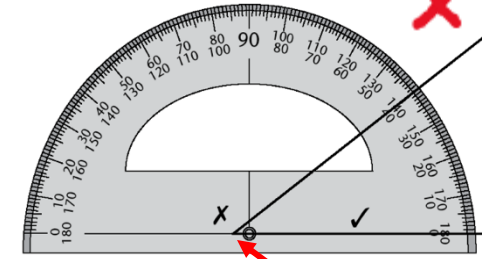
Protractor: This is a piece of equipment used to measure and draw angles.

Discrete Data: Can only have certain values. For example, you cannot have half a person, so data involving people is discrete.

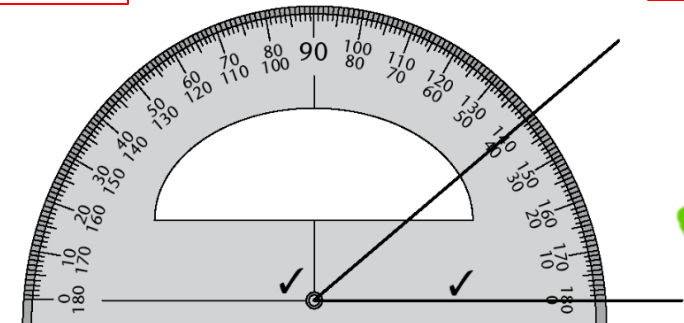
Key Facts: There are **360 degrees** around a point.



Base line is not exactly lined up.



Not exactly in the centre.



origin exactly on top of the vertex and base line exactly on top of arm



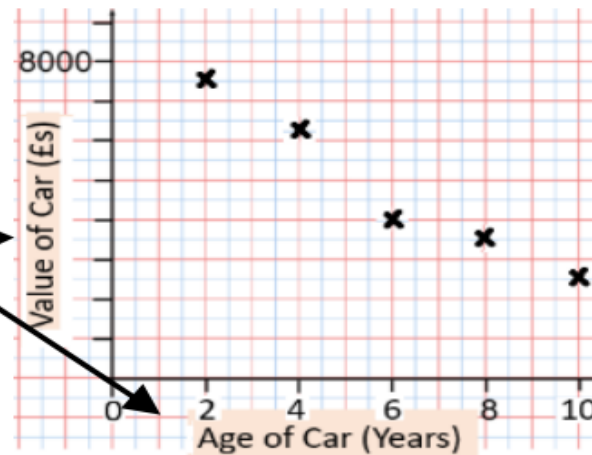
Knowledge Organiser: 3c Scatter Graphs

Draw and interpret a scatter graph.

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

All axes should be labelled



The axis should fit all the values on and be equally spread out

"This scatter graph shows as the age of a car increases the value decreases"

The link between the data can be explained verbally

Key Terms:

Origin – Where two axes meet on a graph. (0,0) point.

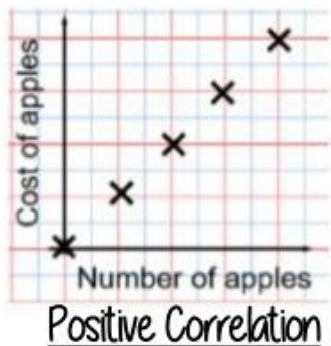
Outlier – A point that lies outside the trend of the graph.

Relationship – The link between two variables e.g. between sunny days and ice cream sales.

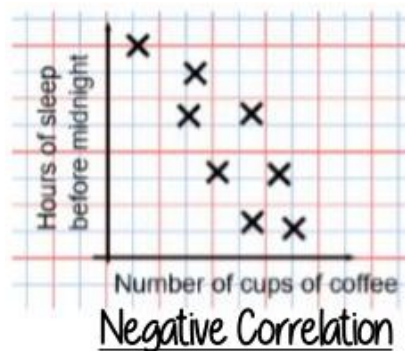
Correlation – The mathematical description for the type of relationship.

Line of Best Fit – A straight line on a graph that represents the relationship between the data on a scatter graph.

Linear Correlation



As one variable increases so does the other variable



As one variable increases the other variable decreases



There is no relationship between the two variables



Knowledge Organiser: 3c Scatter Graphs

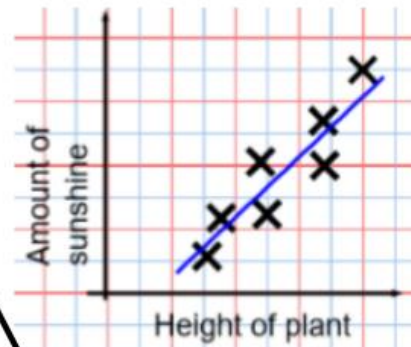
What you need to know:

The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph

Things to know:

- The line of best fit **DOES NOT** need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph



It is only an estimate because the line is designed to be an average representation of the data

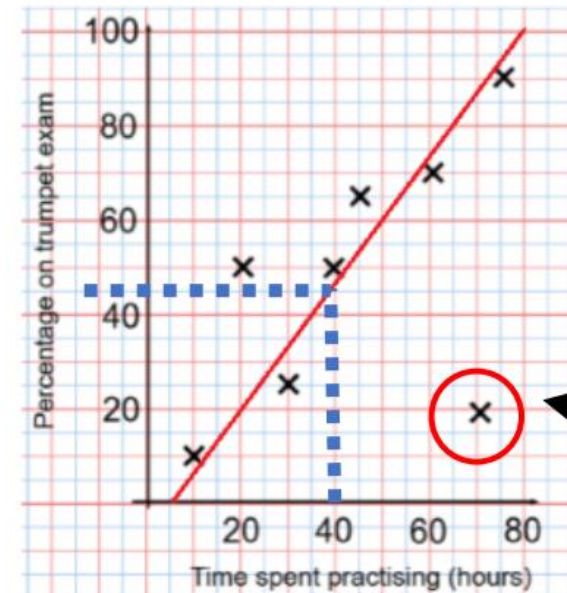
It is always a straight line.

- ✓ The line of best fit should be as close as possible to as many of the points it can but doesn't have to pass through any of them.

Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point.

e.g 40 hours revising predicts a percentage of 45.



Extrapolation is where we use our line of best fit to predict information outside of our data

This is not always useful – in this example you cannot score more than 100%. So revising for longer can not be estimated

This point is an "outlier"
It is an outlier because it doesn't fit this model and stands apart from the data



Knowledge Organiser: 4a Fractions, Decimals and Percentages

What you need to know:

Fractions, decimals and percentages

You need to be able to convert between fractions, decimals and percentages.

Percentages:

$$30\% = 0.30 \text{ or } 0.3 = \frac{30}{100} \text{ or } \frac{3}{10}$$

We sometimes have to simplify our fraction using common times tables.

$$8\% = 0.08 = \frac{8}{100} \text{ or } \frac{4}{50} \text{ or } \frac{2}{25}$$

Percent means out of 100 so this is why we start with 100 as the denominator.

If the decimal has 0 tenths, then it is less than 10%.

Decimals:

$$0.15 = 15\% = \frac{15}{100} \text{ or } \frac{3}{20}$$

$$0.02 = 2\% = \frac{2}{100} \text{ or } \frac{1}{50}$$

Fractions:

$$\frac{45}{100} = 0.45 = 45\%$$

$$\frac{12}{50} \text{ or } \frac{24}{100} = 0.24 = 24\%$$

Unless we know the answer, we must make the denominator 100 then convert.

These are some of the conversions that you need to learn.

Top tips - To convert:

- Percentages to decimals divide by 100.
- Decimals to percentages multiply by 100.
- Percentages to fractions, put over 100.
- Fractions to percentages : make sure the denominator is 100, then the percentage is the numerator.

F	D	P
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%

Key Terms:

Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).

Integer: Whole number.

Ascending Order: Place in order, smallest to largest.

Descending Order: Place in order, largest to smallest.

Percentage: Out of one hundred.

Decimal: A decimal is a fraction written in a special form e.g. 0.6.

You need to be able to:

- Convert between simple fractions, decimals and percentages.
- Convert between fractions and recurring decimals and percentages.
- Compare fractions, decimals and percentages.
- Order fractions, decimals and percentages by converting.



Knowledge Organiser: 4a Fractions, Decimals and Percentages

What you need to know:

Comparing FDP

To be able to compare values, we need to get them in the same format.

Example: Which is bigger 78% or 0.8?

We can either change the 0.8 into a percentage or the 78% into a decimal.

$$0.8 = 80\%$$

Which is bigger 78% or 80%?

80% is bigger so the answer is 0.8.

Make sure you write answer as it was originally written in the question.

Example: Which is bigger $\frac{17}{20}$ or 0.87?

$$\frac{17}{20} = \frac{85}{100} \quad 0.87 = \frac{87}{100}$$

Having them both written out of 100 makes it easier to compare.

Multiply the numerator and denominator by 5 to make the denominator 100.

87 is bigger than 85 so the answer is 0.87.

Ordering FDP

To be able to order FDP, we need to write them all in the same format.

Example: Order from smallest to largest $\frac{1}{4}$ 0.19 0.3 26% $\frac{1}{5}$

You can choose to convert them all into fractions, decimals or percentages as long as you convert them all into the same.

Changing them to percentages:

$$\frac{1}{4} = 25\% \quad 0.19 = 19\% \quad 0.3 = 30\% \quad \frac{1}{5} = 20\%$$

Rewrite the list with the numbers all in the same format.

25%, 19%, 30%, 26%, 20%

From smallest to biggest:

19%, 20%, 25%, 26%, 30%

Answer:

0.19, $\frac{1}{5}$, $\frac{1}{4}$, 26%, 0.3

Make sure you write your answer using the original numbers in the question.



Knowledge Organiser: 4b Percentages

What you need to know:

Percentage of an amount – Non calculator

To calculate any percentage it is useful to start with 10%.

30% of 120: $10\% = 120 \div 10 = 12$

To find 10% we divide by 10.

$30\% = 3 \times 12 = 36$

To find 30% we multiply 10% by 3.

45% of 80: $10\% = 80 \div 10 = 8$ $5\% = 8 \div 2 = 4$

$40\% = 4 \times 8 = 32$

$45\% = 40\% + 5\% = 32 + 4 = 36$

5% is half of 10%
so we divide by 2.

To find 1% we divide the starting amount by 100.

$1\% \text{ of } 30 = 30 \div 100 = 0.3$.

Percentage of an amount – Calculator

When we have a calculator, we can use a multiplier; this is the decimal equivalent of the percentage.

80% of 120: $80\% = 0.80$

$80\% \text{ of } 120 = 0.80 \times 120 = 96$

Change the
percentage to
a decimal and
then multiply.

33% of 90: $33\% = 0.33$

$33\% \text{ of } 90 = 0.33 \times 90 = 29.7$

Be careful if
the percentage
is less than 10.

4% of 88: $4\% = 0.04$

$4\% \text{ of } 88 = 0.04 \times 88 = 3.52$

Take care using
decimal percentages,
still divide by 100.

12.5% of 42: $12.5\% = 0.125$

$12.5\% \text{ of } 42 = 0.125 \times 42 = 5.25$

Key Terms:

Percentage: Out of a hundred.

Decimal: A decimal is a number expressed in the scale of tens, with the digits after the point representing tenths, hundredths, thousandths etc.

Multiplier: This is used to calculate percentages when we have a calculator.

Increase: When an amount goes up.

Decrease: When an amount goes down.

Simple interest: The amount of interest is fixed over a period of time.

Compound interest: The interest earned over time will continue to increase.

You need to be able to:

- Calculate a percentage of an amount.
- Use a multiplier to calculate a percentage of an amount.
- Calculate a percentage increase.
- Calculate a percentage decrease.
- Calculate simple interest.
- Calculate compound interest.



Knowledge Organiser: 4b Percentages

What you need to know:

Percentage increase and decrease

Increase: To calculate a percentage increase we calculate the percentage and add the value on to the original amount.

Non-Calculator: Increase 70 by 65%

$$10\% = 70 \div 10 = 7 \quad 5\% = 7 \div 2 = 3.5$$

$$60\% = 6 \times 7 = 42$$

$$65\% = 60\% + 5\% = 42 + 3.5 = 45.5$$

Calculate 65% by splitting into 10% and 5% and then add the answer on to the original amount.

$$70 + 45.5 = \mathbf{115.5}$$

Calculator: Increase 130 by 26%

$$26\% \text{ of } 130 = 0.26 \times 130 = 33.8$$

$$130 + 33.8 = \mathbf{163.8}$$

Calculate 26% using a multiplier and add this answer onto the original amount.

Decrease: To calculate a percentage decrease we calculate the percentage and subtract the value from the original amount.

Non-Calculator: Decrease 20 by 35%

$$10\% = 20 \div 10 = 2 \quad 5\% = 2 \div 2 = 1$$

$$30\% = 3 \times 2 = 6$$

$$35\% = 30\% + 5\% = 6 + 1 = 7$$

Calculate 35% by splitting into 10% and 5% and then subtract the answer off the original amount.

$$20 - 7 = \mathbf{13}$$

Calculator: Decrease 65 by 14%

$$14\% \text{ of } 65 = 0.14 \times 65 = 9.1$$

$$65 - 9.1 = \mathbf{55.9}$$

Calculate 14% using a multiplier and subtract this answer from the original amount.

Simple interest

To calculate simple interest we start by calculating the percentage and multiplying it by the period of time.

Example: £250 is in a bank account which is paying 5% simple interest per year. How much will be in the bank account at the end of 3 years?

$$5\% = 0.05$$

$$0.05 \times 250 = \pounds 12.50$$

Multiply by 3 because the question asks for 3 years.

$$3 \times \pounds 12.50 = \pounds 37.50$$

Add your answer to the original amount in the question.

$$\pounds 250 + \pounds 37.50 = \pounds 287.50$$

Compound interest

To calculate compound interest we use powers as the amount changes at the end of each year.

Example: £250 is in a bank account which is paying 4% compound interest per year. How much will be in the bank account at the end of 5 years?

$$4\% \text{ increase} = 1.04$$

Interest means an increase so $100\% + 4\% = 104\%$ which as a multiplier is 1.04

$$1.04^5 \times 250 = \pounds 304.16$$

Power of 5 because the questions asks for 5 years.

This is the final answer



Knowledge Organiser: 5a Equations and Inequalities

What you need to know:

Reading and Writing Inequalities

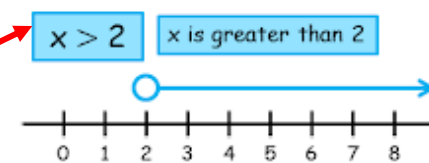
The list of integers for $-2 < x \leq 1$ is -1, 0, 1.

Check the symbols carefully, if they have the line underneath, they include the end value.

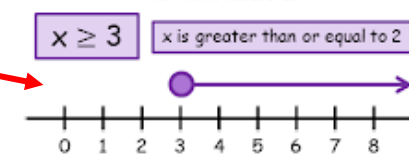
	Greater than	$>$	Greater than or equal to	$\geq$	
	Less than	$<$	Less than or equal to	$\leq$	
	Not equal to	$\neq$			

The arrow points in the same direction as the inequality.

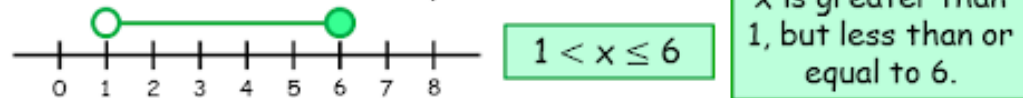
An **open circle** means that the value is **not included**:



A **filled in circle** means that the value is **included**:



If x is **between** two values, use **two circles**:



Solving one step equations/inequalities

To solve any equation or inequality we need to do the inverse of the operation that we see.

$$\begin{array}{rcl} t + 4 & = & 10 \\ -4 & -4 & \\ \hline t & = & 6 \end{array}$$

The inverse of add is subtract and vice versa.

$$\begin{array}{rcl} c - 3 & > & 6 \\ +3 & +3 & \\ \hline c & > & 9 \end{array}$$

$$\begin{array}{rcl} 6y & < & 30 \\ \div 6 & \div 6 & \\ \hline y & < & 5 \end{array}$$

The inverse of multiply is divide and vice versa.

$$\begin{array}{rcl} \frac{m}{7} & = & 4 \\ \times 7 & \times 7 & \\ \hline m & = & 28 \end{array}$$

Key Terms:

Inverse: This is another word for opposite. We complete the opposite operation to the one shown in the question.

Integer: A whole number.

Equation: A mathematical statement that shows that two expressions are equal.

Inequality: They compare two values, showing if one is less than or greater than another value.

Solve: To get the solution or answer to a question.

You need to be able to:

- Read an inequality.
- Represent an inequality on a number line.
- Solve one step equations and inequalities.
- Solve two step equations and inequalities.
- Solve equations and inequalities with brackets.
- Solve equations and inequalities with unknowns on both sides.



Knowledge Organiser: 5a Equations and Inequalities

What you need to know:

Solving two step equations/inequalities

To solve a two-step equation or inequality we need to complete 2 inverse operations in a specific order.

$$6y + 2 = 32$$

$$\begin{matrix} -2 & -2 \end{matrix}$$

Subtract first because the 2 is separate from the y.

$$6y = 30$$

$$\begin{matrix} \div 6 & \div 6 \end{matrix}$$

Divide because it is the inverse of multiplying.

$$y = 5$$

$$\frac{w-5}{3} \geq 4$$

$$\begin{matrix} \times 3 & \times 3 \end{matrix}$$

Multiply first because the entire expression is divided by 3.

$$w - 5 \geq 12$$

$$\begin{matrix} +5 & +5 \end{matrix}$$

Add because it is the inverse of subtracting.

$$w \geq 17$$

Solving equations with brackets

We must expand the bracket first and then solve by doing the inverse of the operations. We use the same method for inequalities.

$$3(2x + 5) = 39$$

Expand brackets first.

$$6x + 15 = 39$$

$$\begin{matrix} -15 & -15 \end{matrix}$$

The inverse of +15 is -15.

$$6x = 24$$

$$\begin{matrix} \div 6 & \div 6 \end{matrix}$$

The inverse of $\times 6$ is $\div 6$.

$$x = 4$$

Solving with unknowns on both sides

To solve an equation or inequality with unknowns on both sides we need to collect all the same terms together, still using inverse operations

$$5x - 20 \leq 3x + 4$$

$$\begin{matrix} -3x & -3x \end{matrix}$$

We subtract 3x from both sides because it is the smaller term of x.

$$2x - 20 \leq 4$$

$$\begin{matrix} +20 & +20 \end{matrix}$$

Then solve like a normal two step equation.

$$2x \leq 24$$

$$\begin{matrix} \div 2 & \div 2 \end{matrix}$$

$$x \leq 12$$

$$2x - 10 = 5x + 2$$

$$\begin{matrix} -2x & -2x \end{matrix}$$

We subtract 2x from both sides because it is the smaller term of x.

$$-10 = 3x + 2$$

$$\begin{matrix} -2 & -2 \end{matrix}$$

$$-12 = 3x$$

$$\begin{matrix} \div 3 & \div 3 \end{matrix}$$

Then solve like a normal two step equation.

$$-4 = x$$

Top tip: Always subtract/add the smaller x terms to avoid getting a negative x term at the end.



Knowledge Organiser: 5b Sequences

What you need to know:

Finding the next term - numbers

When you need to find the next term in the sequence you need to work out what the general rule for the sequence is.

The rule is add 4 because the difference between each number is 4.

1, 5, 9, 13, ... 17
+ 4 + 4 + 4

17 is the next number because $13 + 4 = 17$.

The rule is subtract 7 because the difference between each number is 7.

14, 7, 0, -7, ... -14
-7 -7 -7

-14 is the next number because $-7 - 7 = -14$.

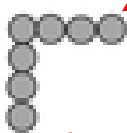
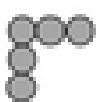
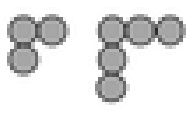
The rule is multiply by 2 because the numbers are doubling.

5, 10, 20, 40, ... 80
x2 x2 x2

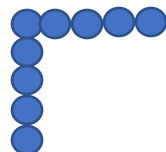
80 is the next number because $40 \times 2 = 80$

Finding the next term - diagrams

This is very similar to continuing a sequence of numbers, we just need to work out what has changed to get from one diagram to the next.



We can see an extra circle has been added along the top and down the side each time. The 4th pattern would look like this.



Key Terms:

Term: Each value in a sequence is called a term.

Rule: The operation you perform to move from one term to the next.

Sequence: A number or picture pattern with a specific rule.

Linear sequence: A sequence that increases or decreases by the same number between each term.

Nth term: A formula which allows you to calculate the term that is in the nth position of the sequence. Also known as the 'position to term' rule.

Generate: When we substitute values into the nth term rule to calculate the terms in the sequence.

You need to be able to:

- Continue a sequence by finding the next term.
- Explain the rule of a sequence verbally and as a written explanation.
- Draw the next diagram in a sequence.
- Find the nth term of a sequence.
- Generate a sequence from the nth term.
- Recognise non-linear sequences.



Knowledge Organiser: 5b Sequences

What you need to know:

Finding the nth term

The nth term is the general rule for a sequence. We can use the nth term to then calculate any term in the sequence.

Here is a sequence: 5, 8, 11, 14, ...

Find the difference between the numbers.

5, 8, 11, 14

+3 +3 +3 = 3n

Remember to calculate how we get from the times table to the original sequence.

3, 6, 9, 12, ...

5, 8, 11, 14, ...

This means that the nth term starts with 3n and we need to look at the 3 times table.

+2

The nth term is $3n + 2$.

Generating a sequence

nth term = $3n - 1$

Substitute $n = 1, 2, 3$ etc for n in the nth term to get the first 3 numbers in the sequence. n is the position in the sequence.

n	$3n - 1$
1	$3 \times 1 - 1 = 2$
2	$3 \times 2 - 1 = 5$
3	$3 \times 3 - 1 = 8$

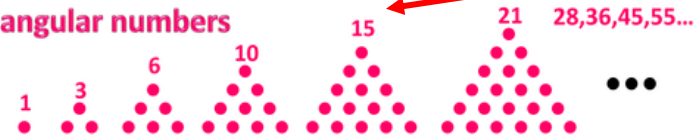
To get the 10th term:
 $3 \times 10 - 1 = 29$.

Sequence = 2, 5, 8, ...

Special sequences

Sometimes sequences do not increase or decrease by a consistent number. These can be quadratic sequences which includes an n^2 term, or they can be other special sequences some of which are shown below,

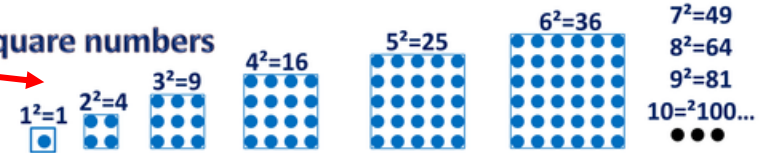
Triangular numbers



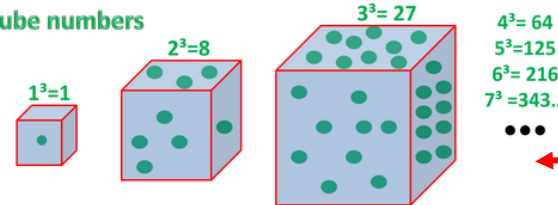
To get from one term to the next you can see that the difference increases by 1 more each time so +2, +3, +4, +5 etc.

These are the square numbers written as a sequence so $1 \times 1, 2 \times 2, 3 \times 3$ etc..

Square numbers



Cube numbers



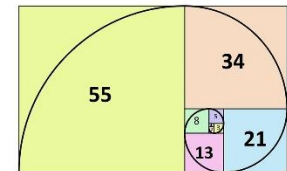
These are the cube numbers written as a sequence so $1 \times 1 \times 1, 2 \times 2 \times 2, 3 \times 3 \times 3$ etc..

The Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377...

The Fibonacci sequence is when we add the 2 previous terms to find the next term.

$1+1=2$
 $1+2=3$
 $2+3=5$
 $3+5=8$
 $5+8=13$
 $8+13=21$
 $13+21=34$
 $21+34=55$
 $34+55=89$
 $55+89=144$
 $89+144=233$
 $144+233=377$

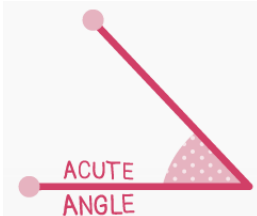




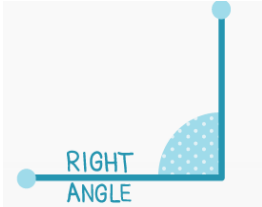
Knowledge Organiser: 6a Parallel Lines and Angle Facts

What you need to know:

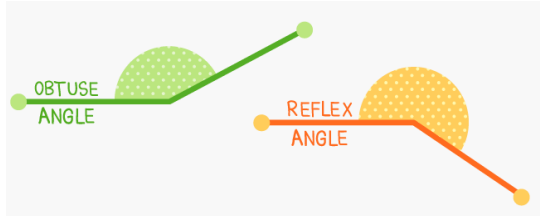
Types of Angles



Less than 90°



90° angle



Bigger than 90° but less than 180°

Bigger than 180° but less than 360°

Types of Lines

Perpendicular Lines

Intersect (cross) at 90°

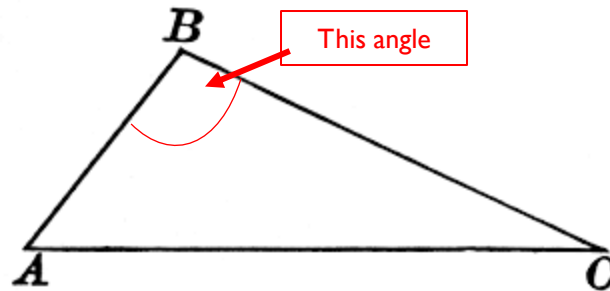
Parallel Lines

Never meet

Angle Notation

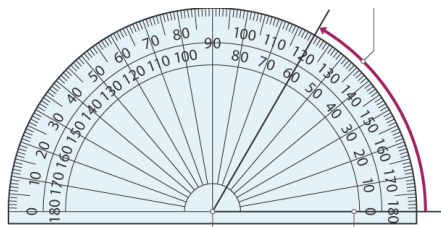
Angles are measured in degrees ($^\circ$).
An angle can be identified like this
The middle letter is the vertex.

$\angle ABC$



Measuring Angles

When measuring angles, make sure that the centre of the protractor is over the **vertex** (corner) of the angle and that the base line of the protractor is along one of the lines of the angle



Always read from zero.
In this example use the inside scale

Ensure the centre and base line are lined up with the angle lines

Key Terms:

Line segment – a line between two points

Point – An exact location.

Intersecting – where two or more lines cross, their common point.

Angle – the amount of turn between two lines and their common point.

Vertically Opposite – angles formed when two or more straight lines cross at a point.

Parallel – always the same distance apart and never touching.

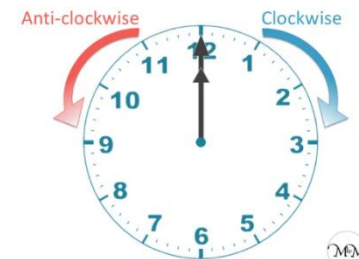
Vertex (plural Vertices) – a corner

Perpendicular – at right angles

You need to be able to:

- Estimate the size of angles;
- Measure angles using a protractor;
- Use letters to identify points, lines and angles;
- Describe angles as turns and in degrees and *understand* clockwise and anticlockwise.
- Know that 360° is a full turn, 180° is a half turn and 90° is a quarter turn.
- Find missing angles using corresponding and alternate angles.
- Understand and use angle properties of parallel lines.

Reminder:





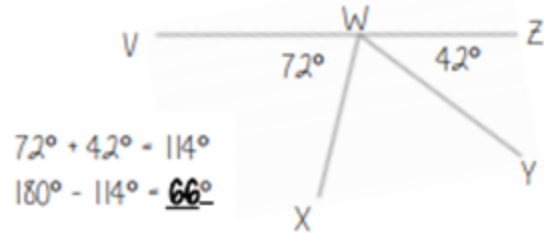
Knowledge Organiser: 6a Parallel Lines and Angle Facts

What you need to know:

Angles on a straight line

Angles on a straight line add up to 180°

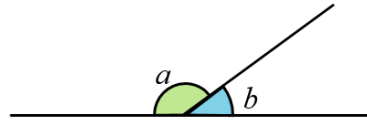
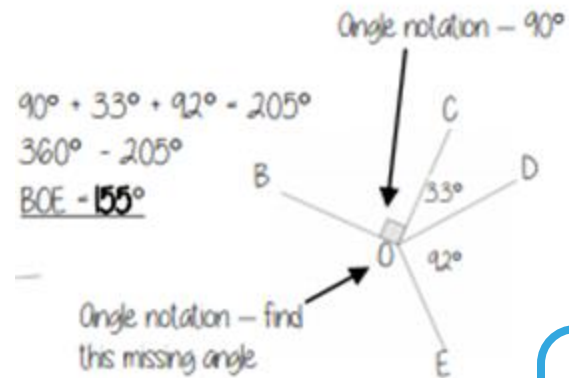
Example – Find angle XWY



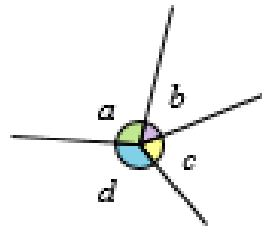
Angles around a point

Angles around a point add up to 360°

Example – Find BOE



$a + b = 180^\circ$
because there are 180° in a half turn.



$a + b + c + d = 360^\circ$
because there are 360° in a full turn.

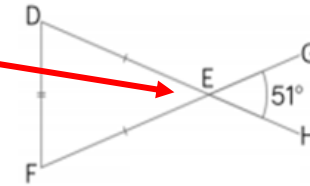
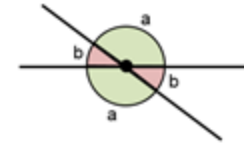
TIP -

Sometimes you will need to use more than one angle fact to solve a problem

Vertically Opposite angles

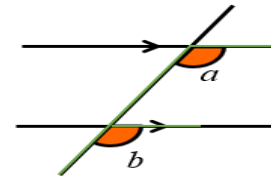
Vertically opposite angles are equal

= 51° because it is vertically opposite



Angles on parallel lines

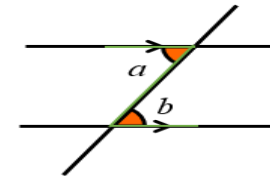
Corresponding angles are equal



$$a = b$$

Look for an F-shape

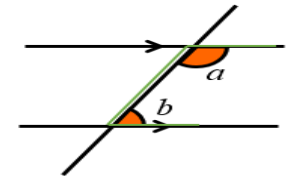
Alternate angles are equal



$$a = b$$

Look for a Z-shape

Interior angles add up to 180°

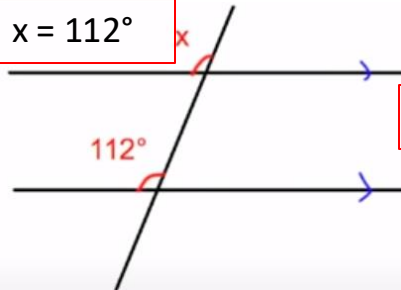


$$a + b = 180^\circ$$

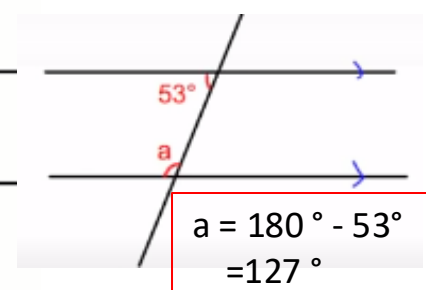
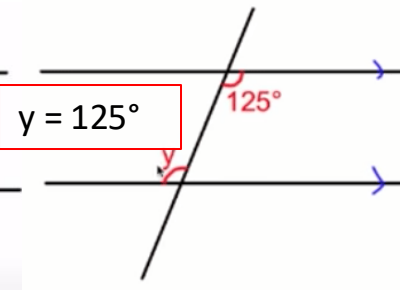
Look for a C- or U-shape

Examples –

$$x = 112^\circ$$



$$y = 125^\circ$$



$$a = 180^\circ - 53^\circ = 127^\circ$$

The 'F' can go in any direction.

The 'Z' can go in any direction.

The 'C' can go in any direction.

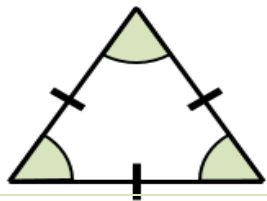


Knowledge Organiser: 6a Triangles and Quadrilaterals

What you need to know:

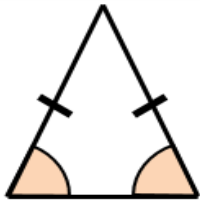
Types of Triangles

Equilateral Triangle



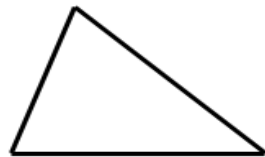
3 equal sides
3 equal angles (60°)
2 sets of parallel sides
3 lines of symmetry

Isosceles Triangle



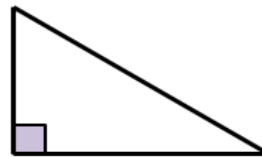
2 equal sides
2 equal angles
1 line of symmetry

Scalene Triangle



0 equal sides
0 equal angles
0 lines of symmetry

Right-angle Triangle

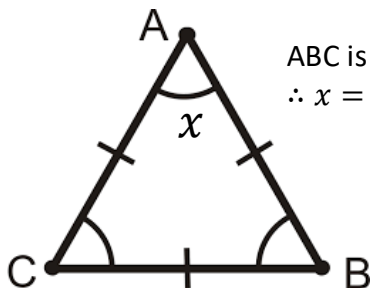
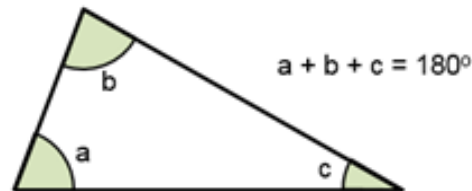


1 right angle

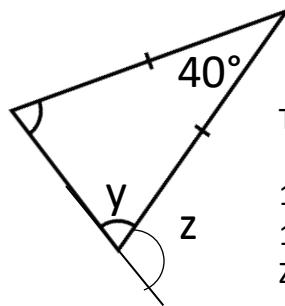
Angles in Triangles

Angles in a triangle add up to 180°

Examples – calculate the missing angles



ABC is an equilateral triangle
 $\therefore x = 180 \div 3 = 60^\circ$



This is an isosceles triangle
 $\therefore$ base angles are equal
 $180 - 40 = 140$
 $140 \div 2 = 70^\circ$ $y = 70^\circ$
 $z = 180 - 70 = 110^\circ$

Tip – work out missing angles in alphabetical order

Key Terms:

Interior Angles: angles inside the shape

Exterior Angles: angle between the side of a shape and a line extended from the adjacent side

Sum: total, add all angles together

Polygon: A 2D shape made with straight lines

Quadrilateral: a four-sided polygon

Convex Quadrilateral: a four-sided polygon where every interior angle is less than 180°

Concave Quadrilateral: a four-sided polygon where one interior angle exceeds 180°

You need to be able to:

- Know the properties of triangles and quadrilaterals
- Group quadrilaterals according to their properties
- Be able to draw different triangles and quadrilaterals
- Complete triangles and quadrilaterals on co-ordinate grids.
- Find missing angles in triangles and quadrilaterals

Reminder:

Plotting coordinates: (x,y)

The x coordinate is how far across you go

The y coordinate is how far up/down you go

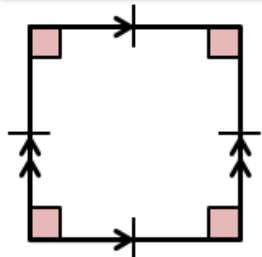


Knowledge Organiser: 6a Triangles and Quadrilaterals

What you need to know:

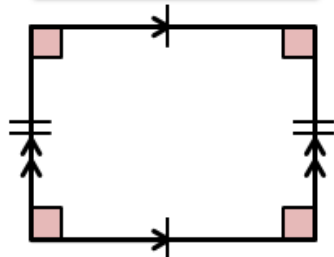
Types of Quadrilaterals

Square



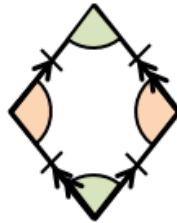
4 equal sides
4 right angles
2 sets of parallel sides
4 lines of symmetry

Rectangle



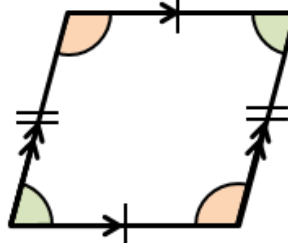
2 sets of equal sides
4 right angles
2 sets of parallel sides
2 lines of symmetry

Rhombus



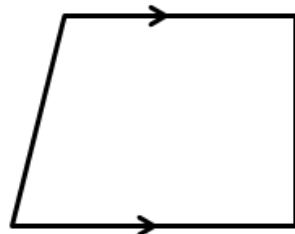
4 equal sides
2 sets of parallel sides
2 pairs of equal angles
2 lines of symmetry

Parallelogram



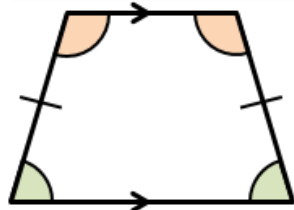
2 sets of equal sides
2 sets of parallel sides
2 pairs of equal angles
0 lines of symmetry

Trapezium



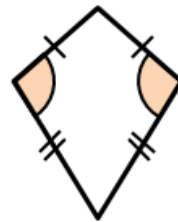
1 set of parallel sides
0 lines of symmetry

Isosceles Trapezium



1 set of equal sides
1 set of parallel sides
2 pairs of equal angles
1 line of symmetry

Kite



2 sets of equal sides
0 parallel sides
1 pair of equal angles
1 line of symmetry

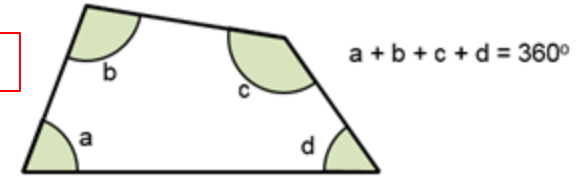
Arrowhead



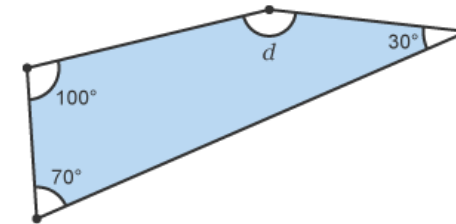
2 sets of equal sides
0 parallel sides
1 pair of equal angles
1 line of symmetry

Angles in Quadrilaterals

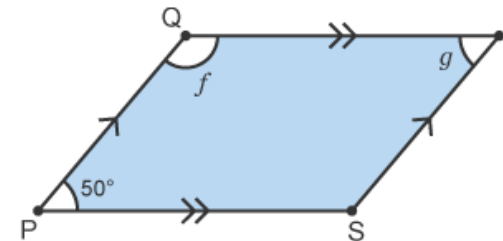
Angles in a quadrilateral add up to 360°



Examples – calculate the missing angles



Angles add up to 360°
 $100 + 70 + 30 + d = 360$
 $200 + d = 360$
 $360 - 200 = d$
 $d = 160^\circ$

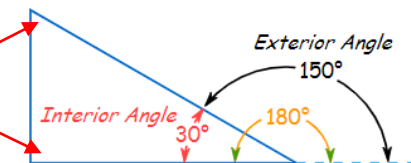


Parallelogram – opposite angles are equal
 $g = 50^\circ$
Using angles in parallel lines $50 + f = 180^\circ$ (co-interior angles)
 $f = 180 - 50$
 $f = 130^\circ$

Interior and Exterior angles

Interior angle + Exterior angle = 180°

These two angles add up to exterior angle. In this example = 150°



Tip – the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices

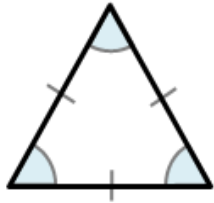


Knowledge Organiser: 6b Interior and Exterior Angles of Polygons

What you need to know:

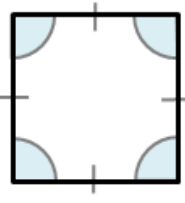
Interior Angles in Regular Polygons

Triangle



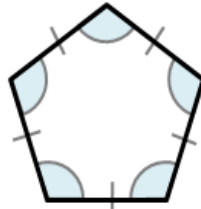
Number of sides	3
Sum of interior angles	180°
Size of each interior angle	60°

Square



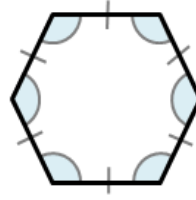
Number of sides	4
Sum of interior angles	360°
Size of each interior angle	90°

Pentagon



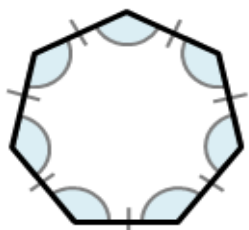
Number of sides	5
Sum of interior angles	540°
Size of each interior angle	108°

Hexagon



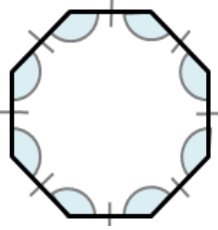
Number of sides	6
Sum of interior angles	720°
Size of each interior angle	120°

Heptagon



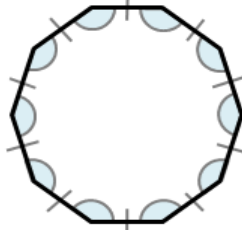
Number of sides	7
Sum of interior angles	900°
Size of each interior angle	128.6° (1dp)

Octagon



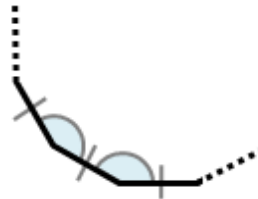
Number of sides	8
Sum of interior angles	1080°
Size of each interior angle	135°

Decagon



Number of sides	10
Sum of interior angles	1440°
Size of each interior angle	144°

n Sided Shape



Number of sides	<i>n</i>
Sum of interior angles	$(n-2) \times 180^\circ$
Size of each interior angle	$\frac{(n-2) \times 180^\circ}{n}$

Key Terms:

Interior Angles - angles inside the shape

Exterior Angles - angles between the side of a shape and a line extended from the adjacent side

Sum - total, add all the angles together

Polygon - a 2D closed shape made with straight lines

Regular - when a shape is regular all sides are the same length and all angles are the same size

Irregular - shape with sides of different lengths and angles of different sizes.

Congruent - objects which are the same shape and size but may be flipped moved or turned.

Tessellation - Shapes tessellate if they fit together to make a pattern with no gaps

You need to be able to:

- Recognise and name different polygons
- Understand the difference between regular and irregular polygons
- Calculate and use the sum of interior angles in polygons
- Know that the sum of the exterior angles of any polygon is 360°
- Know that the interior angle + exterior angle is 180°
- Understand that regular polygons will tessellate if the interior angles divide 360° without remainder

Reminder:

A regular triangle is an **equilateral triangle**

A regular quadrilateral is a **square**



Knowledge Organiser: 6b Interior and Exterior Angles of Polygons

What you need to know:

Exterior Angles

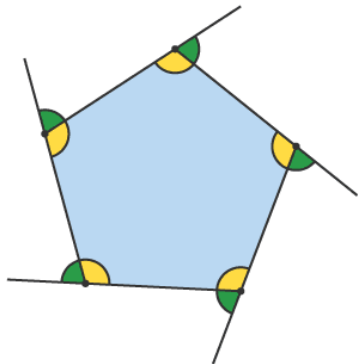
The sum of exterior angles in any polygon is 360°

The size of each exterior angle in a regular polygon is $360^\circ \div \text{number of sides}$

Interior + Exterior Angles

Interior + exterior angle = 180°

Example – calculate the size of each interior and exterior angle of a regular pentagon



$$\begin{aligned}\text{Exterior angle} &= 360 \div 5 = 72^\circ \\ \text{Interior angle} &= 180 - 72 = 108^\circ\end{aligned}$$

Tip – you could also use $\frac{(5-2) \times 180}{5} = 108$ to find the interior angle

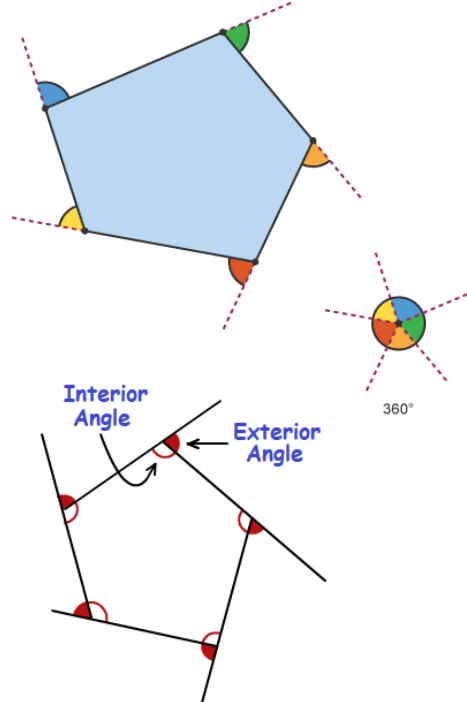
Example – A regular polygon has exterior angles of 20° . How many sides does it have?

$$\text{Exterior angle} = 360 \div \text{number of sides}$$

$$\text{Number of sides} = 360 \div 20 = 18$$

18 sides

$$\text{Rearrange: number of sides} = 360 \div \text{angle}$$



Irregular Polygons

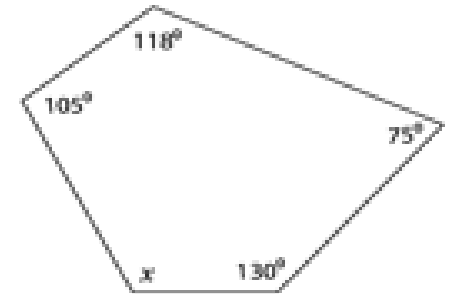
Example – find the value of x

5 sides – irregular pentagon

$$\text{Sum of interior angles} = (5 - 2) \times 180 = 540^\circ$$

$$105 + 118 + 130 + 75 = 428$$

$$540 - 428 = 112 \quad x = 112^\circ$$



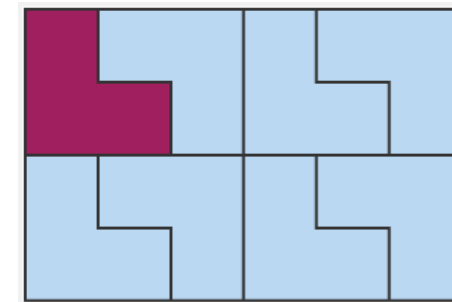
Tessellations

Regular polygons tessellate if the interior angles can be added together to make 360°



Squares, equilateral triangles and hexagons are some examples of shapes that tessellate

Example – tessellate the following shape. Draw at least 5 copies of it



This is just one way that these shapes could tessellate. Remember there should be **no gaps**



Knowledge Organiser: 7 Statistics, Sampling and the Averages

What you need to know:

Averages and range from raw data

Here is a discrete data set. Calculate the mean, mode, median and range for this data.

2 5 3 9 7 7

Mode: 7

Range: $9 - 2 = 7$

Median: 2 3 5 7 7 9

$$\frac{5+7}{2} = 6$$

Put them in order first
Two numbers in the middle – add them together and divide by two

$$\text{Mean: } \frac{2+3+5+7+7+9}{6} = 5.5$$

There are six data values so divide by 6.

Reverse Mean

A hockey team scored the following number of goals in 6 games:

2 3 4 1 0 1

The mean of the goals scored in seven games was 2. How many goals were scored in the seventh game?

Mean = add all together
divide by how values

$$\frac{2 + 3 + 4 + 1 + 0 + 1 + x}{7} = 2 \longrightarrow \frac{11 + x}{7} = 2 \longrightarrow x = 3$$

Solve the equation to find the missing value

Advantages and Disadvantages

Average	Advantage	Disadvantage
Mode	Can be used for qualitative data Easy to obtain	There can be more than one mode or even no mode
Median	Not affected by very large or very small values	Can be time consuming when there is a lot of data
Mean	Takes into account all of the data	Very small or very large values affects the mean

Key Terms:

Mean - Add up the values you are given and divide by the number of values you have.

Median - The median is the middle value, when your data is in order.

Mode - It is the value or item there is the most of.

Range - This is the difference between the largest and smallest values.

Frequency - the number of pieces of data we have.

Grouped Data - If we have a large spread of data, we put it into categories (classes) to make the data easier to display or analyse

You need to be able to:

- Recognise types of data
- Understand sample and population
- Calculate the mean, mode, median and range
- Know the advantages and disadvantages of averages
- Interpret and find averages from frequency tables
- Find the range, modal class, interval containing the median and estimate the mean from grouped data
- Find averages from charts

Reminder:

We looked at interpreting charts in unit 3.
Check you can find averages from different types of charts



Knowledge Organiser: 7 Statistics, Sampling and the Averages

What you need to know:

Averages from Frequency Tables

a) Find the mean of this data

Goals Scored (x)	Frequency (f)	fx
0	2	$0 \times 2 = 0$
1	2	$1 \times 2 = 2$
2	5	$2 \times 5 = 10$
3	1	$3 \times 1 = 3$
Total	10	15

Step 1: calculate the total frequency
 Step 2: calculate $f \times x$
 Step 4: calculate the mean

$$\text{Mean} = \frac{\text{Total } fx}{\text{Total } f}$$

$$\frac{\text{Total } fx}{\text{Total } f} = \frac{15}{10} = 1.5 \text{ goals}$$

b) Find the mode

The mode is the one with the highest frequency

Highest frequency = 5

Mode = 2 goals

c) Find the median

$$\text{Median value} = \frac{\text{Total frequency} + 1}{2}$$

$$\frac{11}{2} = 5.5\text{th value}$$

add the frequency column until you reach the value in-between the 5th and 6th value

Median = 2 goals

d) Find the range

Highest number of goals = 3

$$\text{Range} = 3 - 0 = 3$$

Smallest number of goals = 0

Averages from Grouped Data

a) Estimate the mean of this data

Length (L cm)	Frequency (f)	Midpoint (x)	fx
$0 < L \leq 10$	10	5	$10 \times 5 = 50$
$10 < L \leq 20$	15	15	$15 \times 15 = 225$
$20 < L \leq 30$	23	25	$23 \times 25 = 575$
$30 < L \leq 40$	7	35	$7 \times 35 = 245$
Total	55		1095

Step 1: calculate the total frequency
 Step 2: find the midpoint of each group
 Step 3: calculate $f \times x$
 Step 4: calculate the mean

$$\text{Mean} = \frac{\text{Total } fx}{\text{Total } f}$$

$$\frac{\text{Total } fx}{\text{Total } f} = \frac{1095}{55} = 19.9\text{cm}$$

b) Identify the modal class from this data set

Modal Class is $20 < L \leq 30$

Modal class = the group that has the highest frequency

c) Identify the group in which the median would lie

$$\text{Median value} = \frac{\text{Total frequency} + 1}{2}$$

$$\frac{56}{2} = 28\text{th value}$$

add the frequency column until you reach the 28th value

Median is in the group $20 < L \leq 30$

Tip

For grouped data, the mean can only be an estimate as we do not know the exact values in each group..

Types of data

Qualitative data: data collected that is described in words **not** numbers.
 e.g. race, hair colour, ethnicity.

Quantitative data: this is the collection of numerical data that is either discrete or continuous.

Discrete data: numerical data that is categorised into a finite number of classifications.

e.g. number of siblings in a family, shoe size .

Continuous data: numerical data that can take any value. This data is usually measured on a large number scale.

e.g. height, weight, time, capacity.

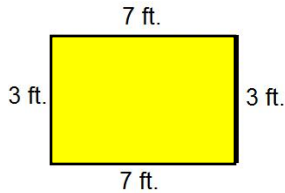


Knowledge Organiser: 8 Perimeter, Area and Volume

What you need to know:

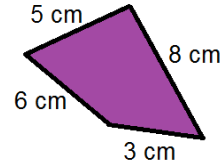
Perimeter

Perimeter: This is the total distance around the outside of the shape.



$7 + 3 + 7 + 3 = 20$
The perimeter is 20 feet.

We must remember to include all of the sides.



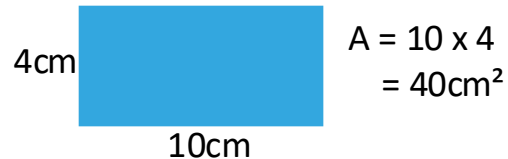
$5 + 8 + 3 + 6 = 22$
The perimeter is 22 cm.

Area

Area: This is the space that a 2D shape takes up inside.

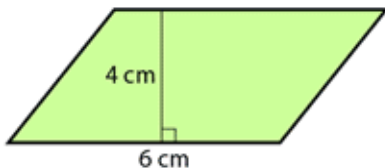
Squares and rectangles:

The formula is the same for both shapes: **Area = Length x Width**



Parallelograms:

The formula is similar to a rectangle but instead of width we use the perpendicular height. **A = Length x Perpendicular Height**



$$A = 6 \times 4 = 24\text{cm}^2$$

Sometimes the length is referred to as the base.

Key Terms:

Perimeter: The total distance around the outside of a shape.

Area: The space inside a 2D shape.

Length: How long a shape is.

Width: How wide a shape is.

Height: How high a shape is.

Base: The bottom of a shape.

Face: The flat part of a 3D solid.

Edge: Where 2 faces meet.

Vertices: Angular points of shapes.

Parallel: Two lines that never meet, always keeping the same distance apart.

Volume: The amount of space that an object occupies.

Capacity: The amount of space that a liquid occupies.

Prism: A prism is a solid object with identical, parallel ends and flat faces. It has the same cross section all along its length.

Cross section: The shape made by cutting through an object, parallel to the base.

You need to be able to:

- Calculate the perimeter of a shape.
- Calculate the area of a square, rectangle, triangle, parallelogram and trapezium.
- Calculate the area of a compound shape.
- Calculate the surface area of a cube and cuboid.
- Calculate the surface area of a triangular prism.
- Identify the number of faces, edges and vertices of a 3D solid.
- Calculate the volume of a cube or cuboid.
- Calculate the volume of a triangular prism.



Knowledge Organiser: 8 Perimeter, Area and Volume

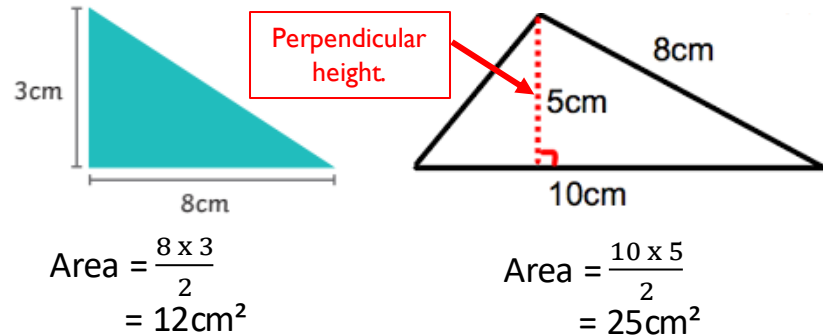
What you need to know:

Area

Triangles: To find the area of a triangle we use the following formula:

$$\text{Area} = \frac{\text{Base} \times \text{perpendicular height}}{2}$$

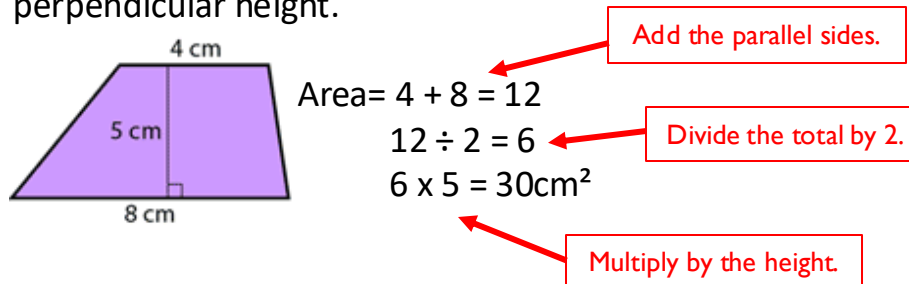
The formula is very similar to a rectangle, but we must divide by 2 because a triangle is half the size of a rectangle with the same base & height.



Trapezium: To find the area of a trapezium we use the following formula:

$$\text{Area} = \frac{(a+b)}{2} \times h$$

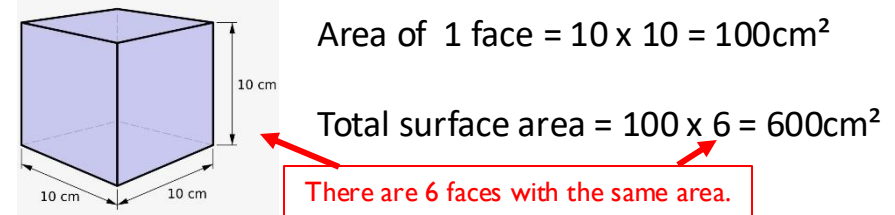
Where a and b are the parallel sides and h is the perpendicular height.



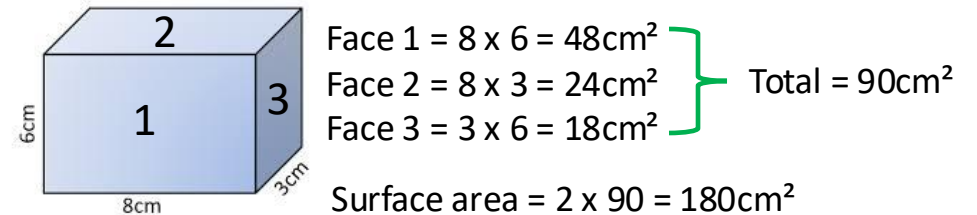
Surface Area

Surface area: This is the area of all of the faces of a 3D solid added together.

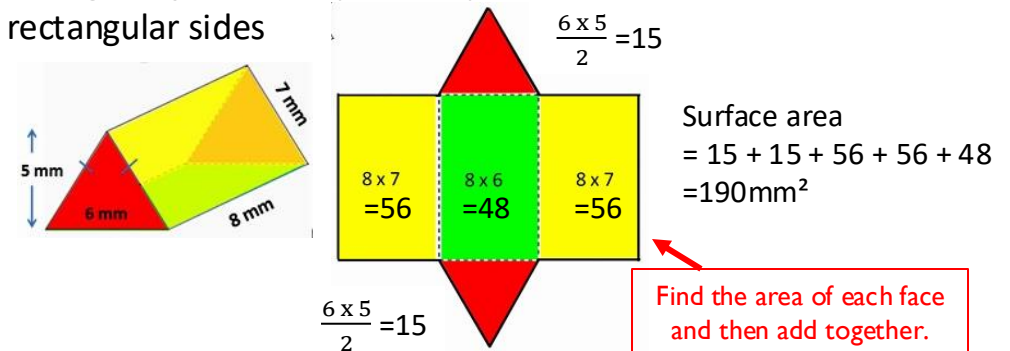
Cubes: Find the area of one of the faces and then multiply by 6. This is because all of the faces of a cube are the same size.



Cuboids: They have 3 pairs of faces. We need to find the area of each of the faces we can see, add them together and then double.



Triangular prism: They have a pair of triangular sides and 3 rectangular sides





Knowledge Organiser: 8 Perimeter, Area and Volume

What you need to know:

3D solids: They have 3 dimensions – length, width and depth.
Here are the main 3D solids that you need to be familiar with.

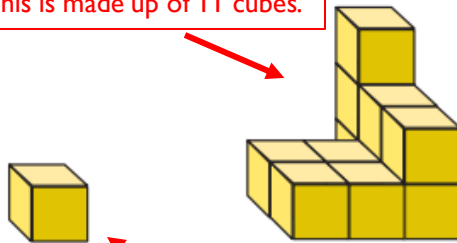
Cube  6 square faces 12 edges 8 vertices	Tetrahedron  4 triangular faces 6 edges 4 vertices	Sphere  1 curved surface 0 edges 0 vertices
Cuboid  6 faces 12 edges 8 vertices	Octahedron  8 faces 12 edges 6 vertices	Triangular prism  5 faces 9 edges 6 vertices
Square-based pyramid  5 faces 8 edges 5 vertices	Cone  1 circular face 1 curved surface 1 curved edge 1 apex	Cylinder  2 circular faces 1 curved surface 2 curved edges 0 vertices

You especially need to know the names of these solids.

Volume

Volume: This is the amount of space that a 3D object occupies.
Sometimes an object is made up of cubes, we can count them to calculate the volume.

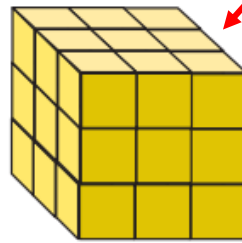
This is made up of 11 cubes.



= 1cm³

This is made up of 1 cube.

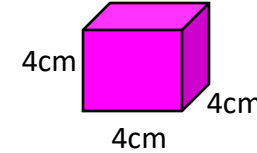
This is made up of 27 cubes.



27cm³

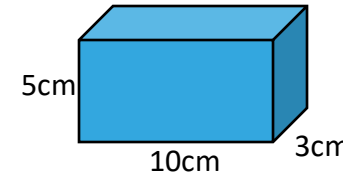
Cubes and cuboids: To calculate the volume of a cube and cuboid we use the following formula:

$$\text{Volume} = \text{Length} \times \text{Width} \times \text{Height}$$



$$\text{Volume} = 4 \times 4 \times 4 = 64\text{cm}^3$$

The units are cubic for volume.



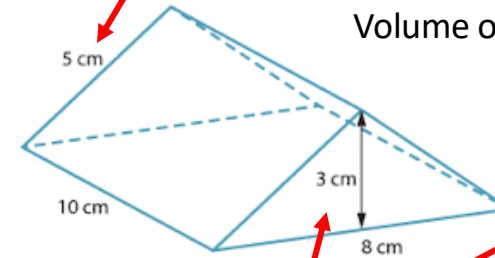
$$\text{Volume} = 10 \times 3 \times 5 = 150\text{cm}^3$$

It doesn't matter which order you multiply in.

Prism: A prism is a solid object with identical ends and flat faces.
The general formula for the volume of a prism is:

$$\text{Volume} = \text{Area of the cross-section} \times \text{Length}$$

We do not need this.



Volume of triangular prism = Area of a triangle x length

$$\text{Area of triangle} = \frac{3 \times 8}{2} = 12$$

$$\text{Volume} = 12 \times 10 = 120\text{cm}^3$$

Area of the cross section.

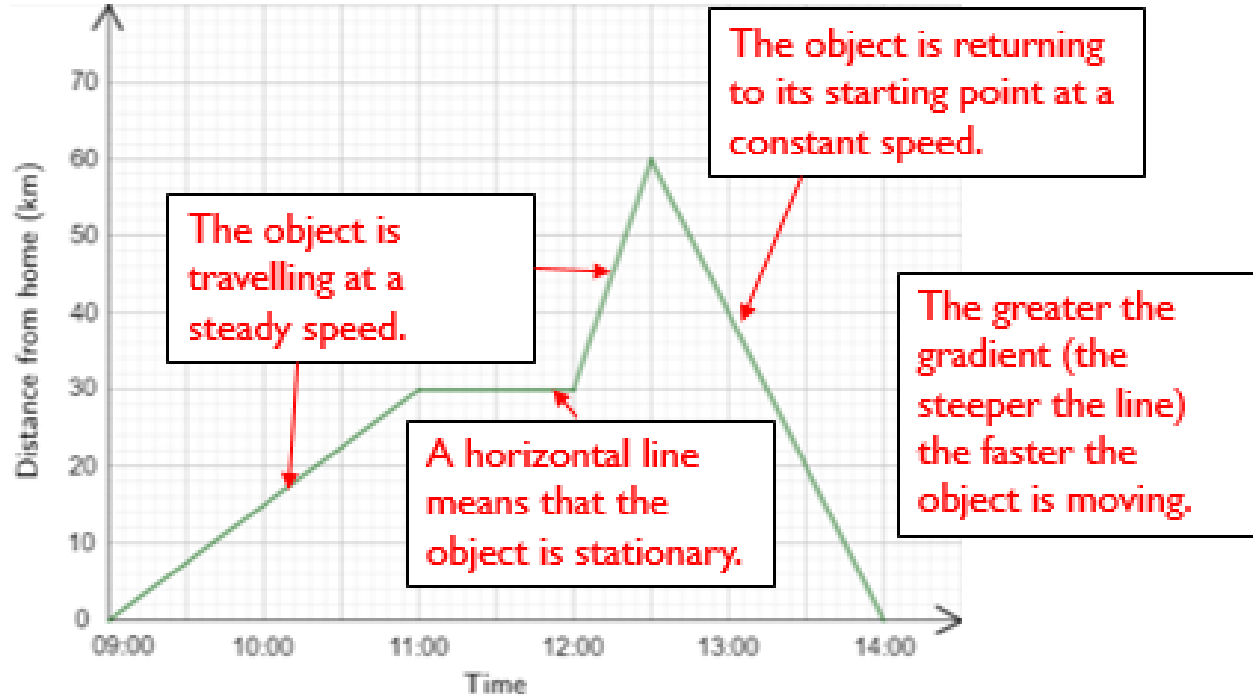
Multiply the area of the cross section by 10 which is the length.



Knowledge Organiser: 9a Real-Life Graphs

What you need to know: Interpret Real-life graphs

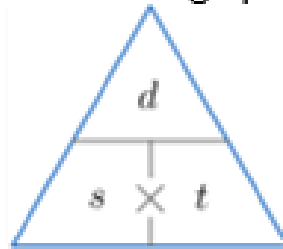
Example 1: Distance/Time Graphs



The speed of an object can be calculated from the gradient of the graph.

E.g. calculate the speed at which the object travelled between 9am and 11am.

$$\begin{aligned}\text{Speed} &= 30 \div 2 \\ &= 15 \text{ km/hr}\end{aligned}$$



Key Terms:

- Journey
- Distance
- Horizontal
- Vertical
- Axis
- Conversion
- Starting point
- Gradient
- Constant
- Speed
- Represents

Key Facts:

- Draw and interpret real life graphs, including distance-time and conversion graphs
- Understand how the vertical axis represents the distance from starting point.
- Understand how the horizontal line on a distance time graph represents an object at rest.
- The gradient of the line represents the speed of the journey



Knowledge Organiser: 9a Real-Life Graphs

What you need to know:

Drawing a conversion graph

You can plot known conversions on a graph to help you to convert other unknown amounts.

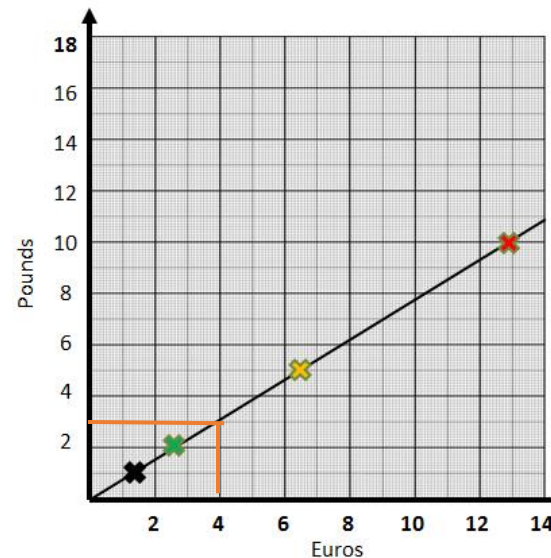
Current exchange rate

£1 = € 1.29

£2 = € 2.58

£5 = € 6.45

£ 10 = € 12.90

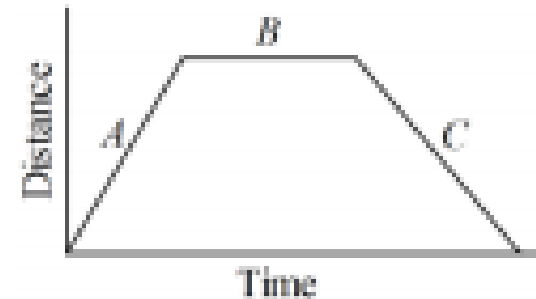


Example 3: Interpreting a conversion graph

Can you use the graph to convert £3 into Euros?

Answer = €3.90

Example 2: Using the graph below, identify what A, B and C mean in terms of travel.

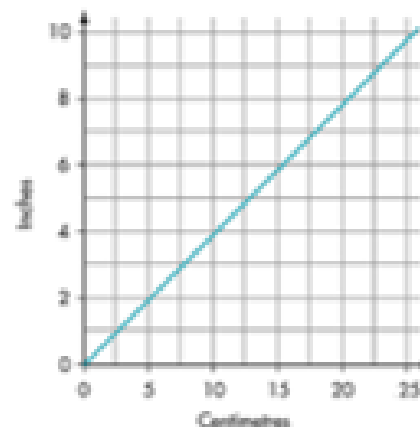


A: Constant speed

B: Stationary

C: Constant speed in the opposite direction to A, back to the start

Using a conversion graph



Conversion graphs can be used to convert between any 2 units which have a linear relationship. Here, you can use the graph to convert between inches and centimetres



Knowledge Organiser: 9b Straight Line Graphs

What you need to know:

Linear Graphs

Linear graphs are straight line graphs. We substitute the x value into the equation to get the y value. Once we have both we can then plot the coordinates and draw the graph.

Draw the graph of $y = 2x - 1$.

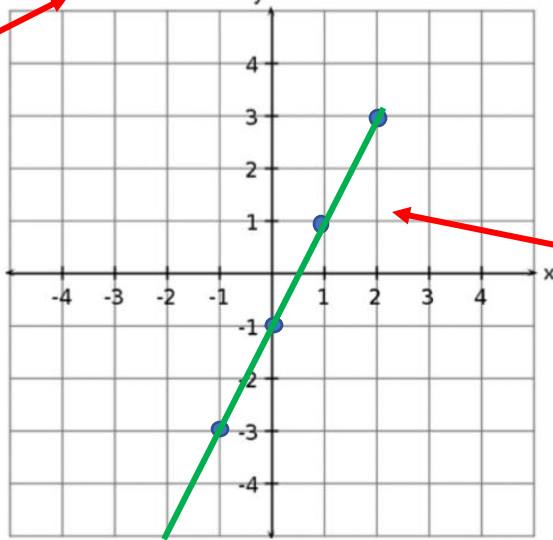
To do this we multiply the x value by 2 and then subtract 1 to get the y value.

$$y = 2x - 1$$

X	-2	-1	0	1	2
Y	-5	-3	-1	1	3

Multiply this value by 2 and then subtract 1 to get the y value.

This coordinate would be (-2,-5).



Don't forget to draw a straight line through all of the coordinates you have plotted.

Notice this graph has a gradient of 2 (the y values go up by 2 each time) and a y-intercept of -1 (the graph cuts through the y axis at -1).

Key Facts:

$$y = mx + c$$

The number next to the x is the **gradient**.

The number on its own is the **y-intercept**.

Key Terms:

Gradient: This describes the steepness of the line.

y-intercept: Where the graph crosses the y-axis.

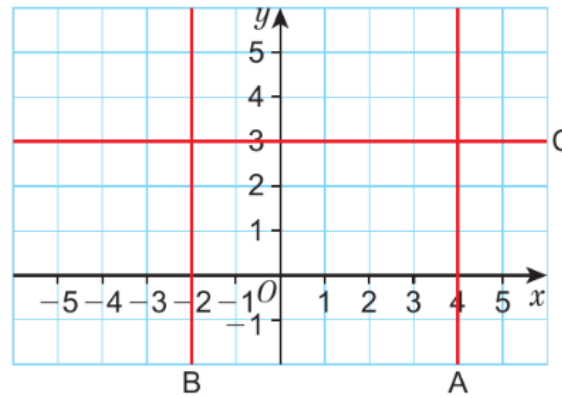
Linear: A linear graph is a straight line.

Parallel: Where 2 lines never meet. They have the same gradient.

Substitute: When a letter is replaced by a number.

Midpoint: The co-ordinates of the middle of a line segment.

Horizontal and Vertical Lines



$$A: x = 4$$

$$B: x = -2$$

$$C: y = 3$$

If a line parallel to the y axis cuts the x axis at 4, the equation is $x = 4$. If the line is parallel to the x axis and cuts the y axis at 3, the equation is $y = 3$.



Knowledge Organiser: 9b Straight Line Graphs

What you need to know:

Equation of a Line

Linear equation: The general equation for a linear (straight line) graph is:

$$y = mx + c$$

m = gradient and c = the y intercept

We need to substitute the gradient first and then substitute one of the coordinates into the general equation to calculate the value of c .

Example:

Find the equation of the line with gradient 5 going through (3, 4).

Replace m with the gradient 5.

$$y = mx + c$$

$$y = 5x + c$$

Substitute in (3, 4)

$$4 = 5 \times 3 + c$$

$$4 = 15 + c$$

Solve to find c .

$$-11 = c$$

Put c into the general equation.

$$\text{Equation: } y = 5x - 11$$

Substitute the coordinates into the general equation. $x=3$ and $y=4$.

Identifying the Gradient and Intercept

The equations of all straight lines can be written in the form:

Gradient – The number in front of the x .
This tells us how steep the line is.

$$y = mx + c$$

Intercept – The number on its own.
This is where the line cuts the y axis.

Example: Find the gradient and intercept of the following lines.

1) $y = 5x - 2$

Grad = 5 Intercept = - 2

2) $2y = 4x + 5$

$$y = 2x + 2.5$$

Grad = 2 Intercept = 2.5

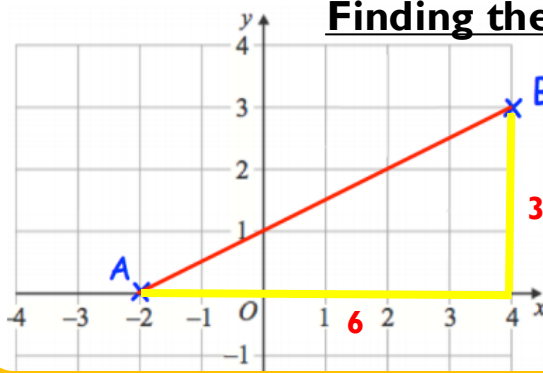
3) $x + y = 10$

$$y = -x + 10$$

Grad = - 1 Intercept = 10

Rearrange all equations so they are in the form $y = mx + c$ (the y must be isolated)

Finding the Gradient From a Graph



A and B are 'good' points as they lie on exact coordinates.

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{3}{6} = \frac{1}{2}$$

So the gradient of this line is $\frac{1}{2}$

Parallel Lines

Parallel lines: The gradient of parallel lines is the same, this is why they never meet.

$$y = 2x + 1$$

$$y = 2x - 4$$

$$y = 2x$$

The gradients are all 2 here so they are all parallel.

Midpoint

Example: Midpoint of (2,4) and (5,6)

$$\frac{2 + 5}{2} = 3.5$$

$$\frac{4 + 6}{2} = 5$$

Answer:
(3.5, 5)

Add the x coordinates and divide by 2. Add the y coordinates and divide by 2.



Knowledge Organiser: 10 Transformations

What you need to know: A transformation is a way of changing the size or position of a shape.

There are 4 types of **Transformations**:

Reflection

Rotation

Translation

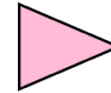
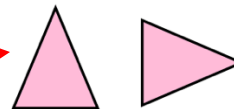
Enlargement

Congruent and similar shapes

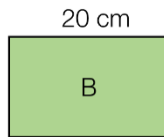
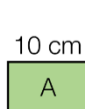
Congruent: This is when the shapes are an identical shape and size.



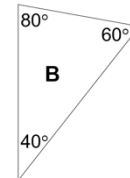
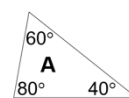
These are pairs of congruent shapes because they are the same shape and size.



Similar: This is when the shapes have the same size angles, or one is an enlargement of the other.

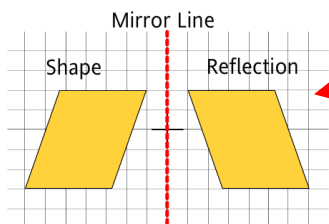


These are pairs of similar shapes because one is an enlargement of the other.



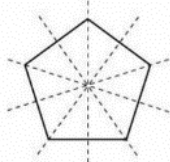
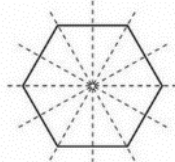
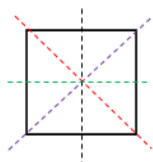
Reflection

A transformation in which an object is reflected across a line, creating a mirror image.



The distance from the mirror line needs to be the same for the reflection as the original shape.

Reflectional symmetry: A type of symmetry where one half of an image is the reflection of the other half. You can have many lines of symmetry.



Key Terms:

Transformation: This means something about the shape has 'changed'.

Reflection: A shape has been flipped using a mirror line.

Reflectional Symmetry: A type of symmetry where one half of an image is the reflection of the other half.

Rotation: A shape has been turned.

Translation: A movement of a shape using a vector.

Enlargement: A change in size, either bigger or smaller.

Congruent: These shapes are the same shape and same size but can be in any orientation.

Similar: Two shapes are mathematically similar if one is an enlargement of the other.

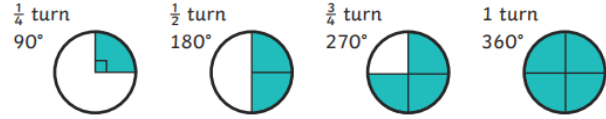


Knowledge Organiser: 10 Transformations

What you need to know: A transformation is a way of changing the size or position of a shape.

Rotation

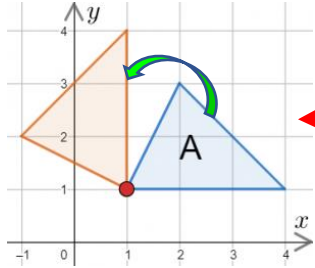
The size does not change but the shape is turned around a point. We must use tracing paper. We need to rotate using a given number of degrees.



We need the:

- Centre
- Number of degrees
- Direction

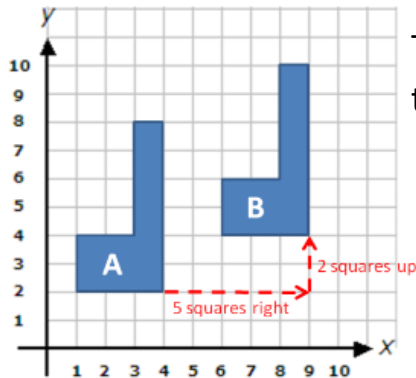
Rotate shape A anti-clockwise about (1,1).



We need to put our pencil on this centre point to complete the rotation with our tracing paper.

Translation

A transformation is where every point of a shape moves the same distance in the same direction. The distance and direction is specified by a vector.



The vector to go with this transformation is $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

This means 2 spaces up.

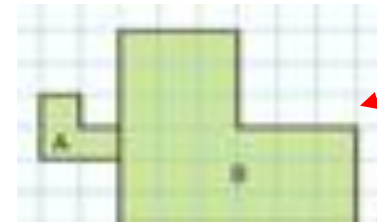
Top = Left (-) Right (+)
Bottom = Down (-) Up (+)

This means 5 spaces to the right.

Enlargement

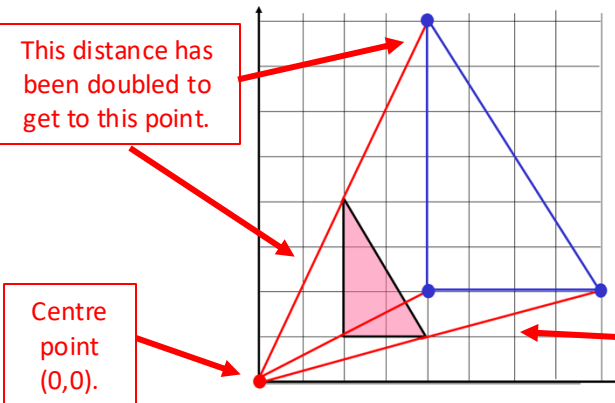
An enlargement is when a shape changes in size by using a scale factor. The scale factor can make a shape bigger or smaller. A scale factor of 2 = shape doubles in size, a scale factor of $\frac{1}{2}$ would halve the size.

Enlarge shape A by scale factor 3.



Scale factor 3 –
Multiply each side length by 3.
It does not matter where you draw it.

From a centre: Enlarge shape A, scale factor 2, centre (0, 0). It is important that you answer for this question is in a specific place because it is from a centre point.



Scale factor 2 –
Double the distance between each vertex (corner) and the centre of enlargement.



Knowledge Organiser: 11a Ratio

What you need to know:

Writing a Ratio

Ratio: The is the relationship between two or more numbers and each number is separate by a colon.



The ratio of footballs to rugby balls: 1:4

The ratio of rugby balls to footballs: 4:1

Football is mentioned first so that is why the 1 comes before 4.

Rugby is mentioned first so that is why the 4 comes before 1.

As fractions: If we wanted to represent the ratio as fractions then:

1 : 4

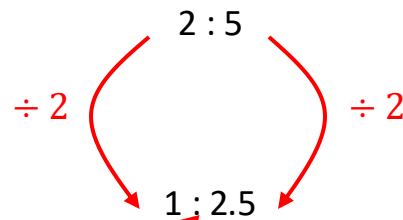
$= \frac{1}{5} : \frac{4}{5}$

The denominator comes from adding the two parts of the ratio together.

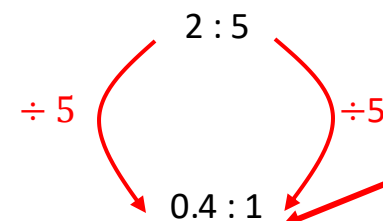
Writing ratios as 1:n or n:1

This means that the ratio needs to be simplified in a specific way. You may end up with fractions or decimals as part of your answer.

Write 2: 5 in the form 1 : n



Write 2: 5 in the form n : 1



You must end up with a 1 in the correct place - read the question carefully!

Key Terms:

Ratio: Relationship between two or more numbers.

Part: This is the numeric value '1' of, would be equivalent to.

Simplify: Divide all parts of a ratio by the same number.

Equivalent: Equal in value.

Convert: Change from one form to another.

Scale: The ratio of the length in a drawing to the length of the real thing.

Proportion: A name we give to a statement that two ratios are equal.

Exchange rate: The value of one currency for the purpose of conversion to another.



Knowledge Organiser: 11a Ratio

What you need to know:

Simplifying a ratio

You must make sure that your ratio has been simplified fully by finding the highest common factor.

Simplify $12 : 20$
 $\div 4$
 $= 3 : 5$

This could have been done in two steps by dividing by 2 and then by 2 again.

Simplify $60 : 40 : 100$
 $\div 10$
 $= 6 : 4 : 10$
 $\div 2$
 $= 3 : 2 : 5$

This could have been done in one step by dividing by 20.

Sharing in a ratio

Sharing in a ratio: To share in a ratio we can use bar modelling to visualise the steps.

Add the parts of the ratio together.

Share £45 in the ratio 2:7.

$2 + 7 = 9$ shares

Divide the total by the number of equal shares.

$45 \div 9 = 5$

$2 : 7$ x

Multiply each part of the ratio by the value of one share.

$= £10 : £35$

2 : 7

5	5
5	5
=10	5
	5
	5
	5
	5
	=35

Sharing ratio when given the value of one share:

Joy and Martin share money in the ratio 2:5. Martin gets £18 more than Joy. How much do they each get?

$5 - 2 = 3$ shares difference

$£18 = 3$ shares

$18 \div 3 = 6$

$£6 = 1$ share

$2 : 5$ x

Find how many parts difference there are and then divide.

2 : 5

6	6
6	6
	6
	6
	6
	6
	=30

=12

$= £12 : £30$ Joy has £12, Martin has £30

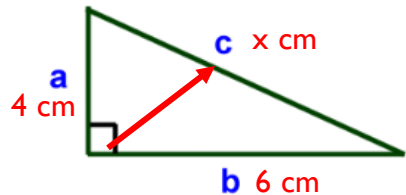


Knowledge Organiser: 12 Pythagoras' Theorem and Trigonometry

What you need to know:

Pythagoras' Theorem - Hypotenuse

You should always label the hypotenuse first.
This is the side facing the right angle.



This is surd form.
Sometimes you will be asked to leave your answer like this.

$$a^2 + b^2 = c^2$$

1) Substitute your values into the formulae:

$$4^2 + 6^2 = x^2$$

2) Work out the values that you can.

$$16 + 36 = x^2$$

$$52 = x^2$$

3) Now use inverse operations to isolate x.

$$52 = x^2$$

$$(\sqrt{}) \quad (\sqrt{})$$

$$\sqrt{52} = x$$

$$7.211102551 \text{ cm} = x \text{ or } 7.21 \text{ to 3 s.f}$$

Pythagoras' Theorem – Shorter Sides

$$a^2 + b^2 = c^2$$

Sometimes you are asked to calculate the shorter sides, see below.

1) Substitute your values into the formulae:

$$10^2 + x^2 = 14^2$$

2) Work out the values that you can.

$$100 + x^2 = 196$$

3) Now use inverse operations to isolate x.

$$100 + x^2 = 196$$

$$(-100) \quad (-100)$$

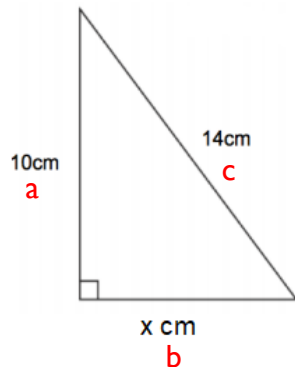
$$x^2 = 96$$

$$(\sqrt{}) \quad (\sqrt{})$$

$$\sqrt{96} = x$$

$$x = 9.797958971 \text{ cm or } 9.80 \text{ cm to 3 s.f}$$

You need to get the numbers on one side, the x on it's own.
An extra step is needed.



Key Terms:

Hypotenuse: The longest side in a right angled triangle.

Opposite: The side facing the angle in a right angled triangle.

Adjacent: The side next to the angle given in a right angled triangle.

Square number: The result when you multiply a number by itself.

Inverse operation: The operation that reverses the effect of another operation.

Sine, Cosine, Tangent: Trigonometric ratios, relating to buttons on the calculator.

You need to be able to:

- Identify the hypotenuse in a right angled triangle.

- Use Pythagoras' theorem to find the hypotenuse and shorter sides.

- Use trigonometry to find lengths and angles in right angled triangles.

- Recall exact trigonometric values.

Reminder:

Square numbers: This is when we multiply a number by itself.

$$1 \times 1 = 1$$

$$2 \times 2 = 4$$

$$3 \times 3 = 9 \text{ etc}$$

Square roots: This is the number that we started with to get the square numbers.

$$\sqrt{49} = 7 \text{ because } 7 \times 7 \text{ is } 49$$

$$\sqrt{100} = 10 \text{ because } 10 \times 10 \text{ is } 100$$

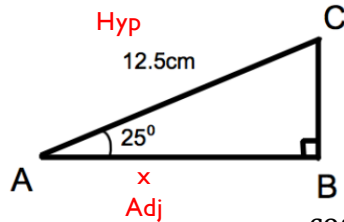


Knowledge Organiser: 12 Pythagoras' Theorem and Trigonometry

What you need to know:

Trigonometry – Finding a side 1

Calculate the length of AB.



- Step 1 – Label the sides you need as O, A or H.
 Step 2 – Use this to decide which trig ratio to use.
 Step 3 – Substitute the given values into the formula.
 Step 4 – Use inverse operations to rearrange & isolate x.

$$\cos(25) = \frac{x}{12.5}$$

$$\cos(25) \times 12.5 = x$$

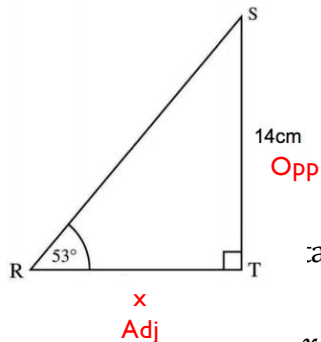
$$x = 11.32884734 \text{ cm}$$

Don't round your answer, you get no marks for this!

The inverse of dividing by 12.5 is multiplying.

Trigonometry – Finding a side 2

Calculate the length of RT.



- Step 1 – Label the sides you need as O, A or H.
 Step 2 – Use this to decide which trig ratio to use.
 Step 3 – Substitute the given values into the formula.
 Step 4 – Use inverse operations to rearrange & isolate x.

$$\tan(53) = \frac{14}{x}$$

$$x = \frac{14}{\tan 53}$$

$$x = 10.5497567 \text{ cm}$$

When the unknown value is on the bottom of the fraction the x and $\tan(53)$ swap places.
 This is because you have multiplied the LHS by x and then divided the RHS by $\tan(53)$.

SOH

$$\text{SINE} = \frac{\text{OPP}}{\text{HYP}}$$

CAH

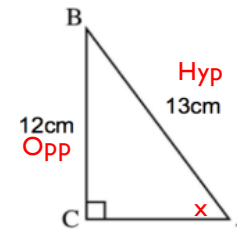
$$\text{COSINE} = \frac{\text{ADJ}}{\text{HYP}}$$

TOA

$$\text{TANGENT} = \frac{\text{OPP}}{\text{ADJ}}$$

Trigonometry – Finding an angle

Calculate the size of angle BAC.



- Step 1 – Label the sides you need as O, A or H.
 Step 2 – Use this to decide which trig ratio to use.
 Step 3 – Substitute the given values into the formula.
 Step 4 – Use inverses to rearrange & isolate x.

$$\sin(x) = \frac{12}{13}$$

$$x = \sin^{-1}\left(\frac{12}{13}\right)$$

$$x = 67.38013505^\circ$$

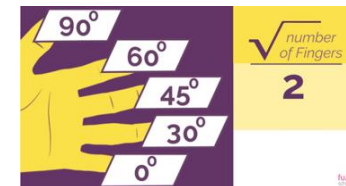
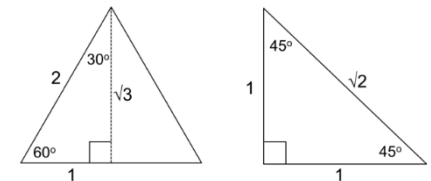
I have not labelled the third side as it has no information written on and I am not trying to calculate it.

The inverse of sin, cos and tan are $\sin^{-1}$, $\tan^{-1}$, $\cos^{-1}$. They are found by pressing shift sin on your calculator.

Trigonometry – Exact values

For your exam you will need to learn the following values.
 (Use the hand trick or triangles to help you learn them)

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	–



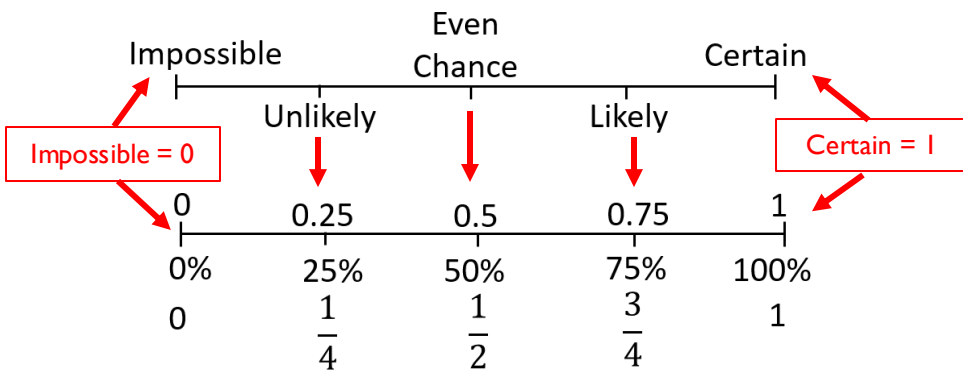


Knowledge Organiser: 13 Probability

What you need to know:

Probability scales

Probability is measured between 0 and 1; it cannot be more than 1. Below are examples of probability scales with the keywords and numerical values.



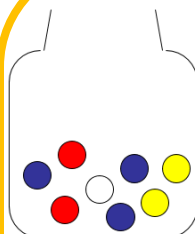
Probabilities can be written as:

- Fractions
- Decimals
- Percentages

Key Fact:

Probabilities add to 1. Probabilities will always be between 0 and 1.

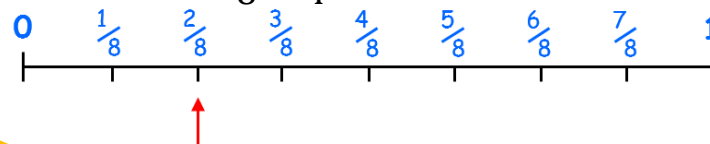
Writing as fractions



Calculate the probability that a bead chosen will be **yellow** and then show your answer on the probability scale.

$$\text{Probability} = \frac{\text{Number of yellow outcomes}}{\text{Total number of outcomes}}$$

$$P(\text{Yellow}) = \frac{2}{8} = \frac{1}{4}$$



Expected outcomes

If we complete an experiment a given number of times how many times we would expect an event to occur.

If the probability of picking a green counter was $\frac{2}{5}$ and we picked a counter out 40 times then we would expect to get green 16 times.

$$\frac{2}{5} \times 40 = \frac{2}{5} \text{ of } 40 = 16$$

Probability $\times$ number of times the experiment is completed.

Key Terms:

Probability: The chance of something happening as a numerical value.

Outcome: This is a possible result of the experiment.

Impossible: The outcome cannot happen.

Certain: The outcome will definitely happen.

Even chance: There are two different outcomes each with the same chance of happening.

Expectation: The amount of times you expect an outcome to happen based on probability.

Two-way table: A table of data linked to 2 variables.

Venn diagram: A Venn diagram shows the relationship between a group of different things (a set) in a visual way.

Probability tree: A diagram used to show and help to calculate, all of the possible outcomes.

Experimental probability: The probability based on the results of an experiment.

Theoretical probability: The probability of what we expect to happen.



Knowledge Organiser: 13 Probability

What you need to know:

Theoretical Probability

Theoretical: This is the probability that an outcome will occur using reasoning or by completing a calculation. It is the expected outcome.

5 appears.

$$P(\text{roll a 5}) = \frac{1}{6}$$



Roll a die once

Theoretical Probability

Number of favorable (desired) outcomes
Total number of possible outcomes

How many 5's?

How many number in total?

This is an example of theoretical probability because we know that there are 6 numbers on a dice and only one of them is a 5.

Experimental Probability

Experimental: This refers to the probability of an event occurring when an experiment was conducted.

5 appears 140 times.

$$P(\text{roll a 5}) = \frac{140}{1000}$$



Roll a die 1000 times.

Number of times the event occurred
Number of trials

How many 5's?

How many times did you do the experiment?

This is an example of experimental probability because we have completed the experiment of rolling the dice. We got 140 5's out of 1000 rolls of the dice.

Sample Space Diagrams

The sample space below records all the possible outcomes when **2 four sided spinners** are spun, and their **totals multiplied** together.

This will not always be a multiplication. It could be any operation so make sure you check!

x	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Probability From a Table

To calculate the probability from a table like the one seen below we need to:

1. Add the probabilities we know together.
2. Subtract our total from 1.

Letter	A	B	C	D	E
Probability	0.07	0.15	0.26		0.18

$$0.07 + 0.15 + 0.26 + 0.18 = 0.66$$
$$1 - 0.66 = 0.34$$

Remember that probabilities must add up to 1.

The probabilities for the different parts of a 4 sided biased spinner are shown in the table below.

Number	1	2	3	4
Probability	0.1	0.2	0.4	

Find the missing probability

The spinner is spun 100 times.
Calculate an estimate for the number of times the spinner will land on number 1

$$0.1 + 0.2 + 0.4 = 0.7$$
$$1 - 0.7 = 0.3$$

Estimate of a 1: $100 \times 0.1 = 10$.

Multiply the probability of the outcome by the number of times the spinner is spun.



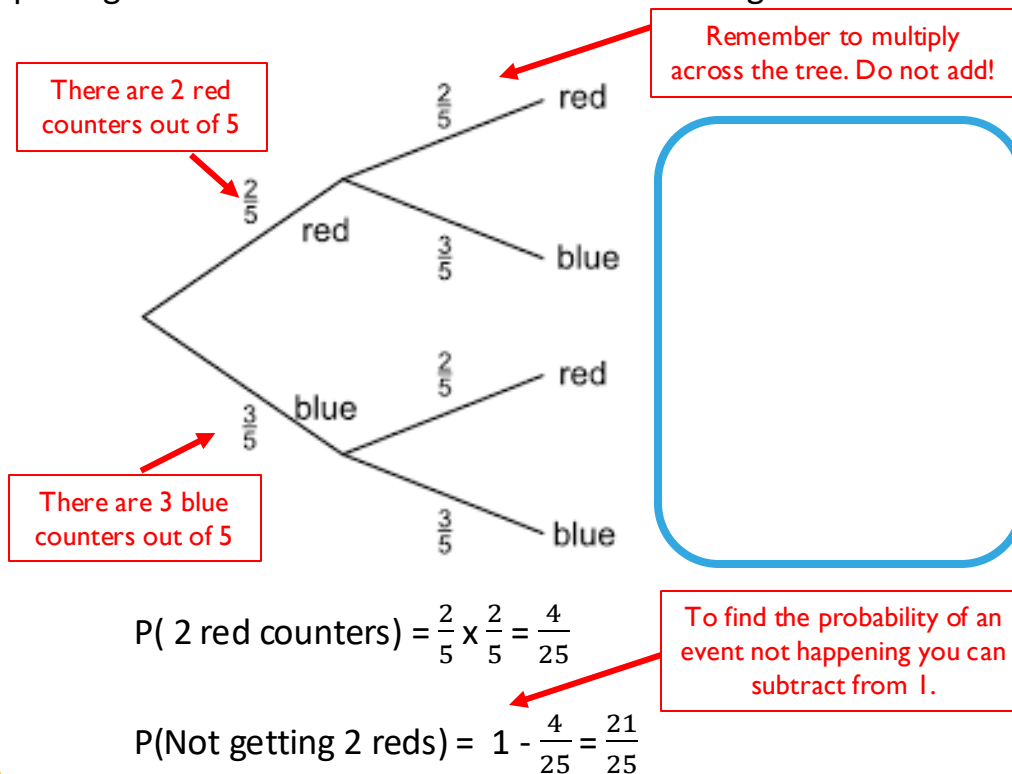
Knowledge Organiser: 13 Probability

What you need to know:

Probability trees

Probability trees: These are a way of recording the outcomes of multiple events and calculating their probability. To find the probability of an event we multiply across the branches to calculate the probabilities of each outcome.

Here we have a probability tree that shows the probability of picking a red counter or blue counter out of a bag.



Two-way tables

Two-way tables are a way of sorting data so that the frequency of each category can be seen quickly and easily.

The two-way table shows some information about the number of students in a school.

	Year Group			Total
	9	10	11	
Boys	149	133	125	407
Girls	154	123	147	424
Total	303	256	272	831

$$831 - 407 = 424$$

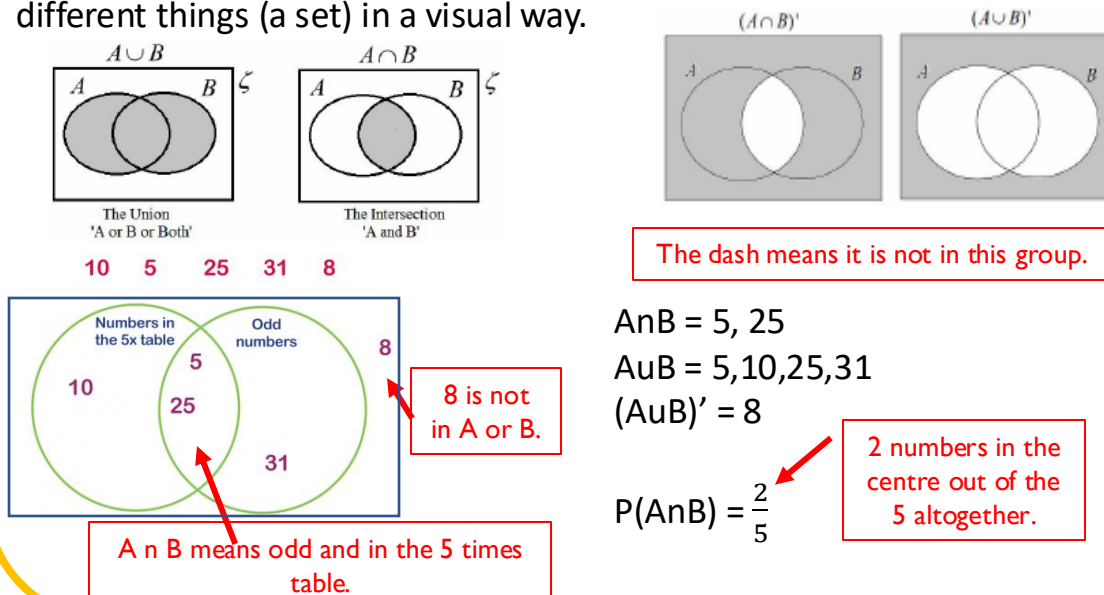
$$407 - (125 + 133) = 149$$

$$256 - 123 = 133$$

$$P(\text{year 9 girl}) = \frac{154}{831}$$

Venn diagrams

A Venn diagram shows the relationship between a group of different things (a set) in a visual way.

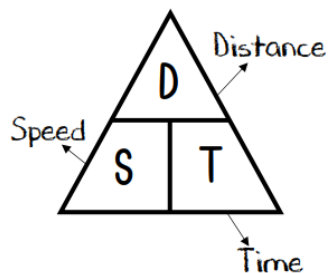




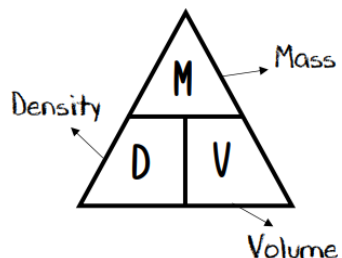
Knowledge Organiser: 14 Multiplicative Reasoning

What you need to know:

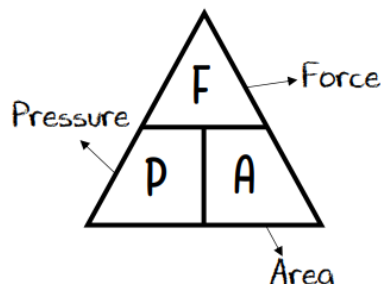
Average Speed



Density



Pressure



Compound Measures

Speed Example:

A car travels 90 kilometres in 2 hours 15 minutes. What is the average speed in km/h?

Use $\text{speed} = \frac{\text{distance}}{\text{time}}$

Answer in km/h

Distance = 90 km

Time = 2 hours 15 minutes = 2.25 hours

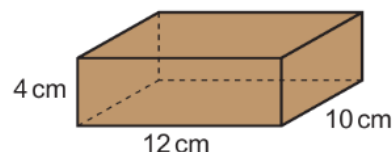
Average speed = $\frac{90}{2.25} = 40 \text{ km/h}$

Convert to a decimal to make the calculation easier.

Substitute values into the formula.

Density Example:

The diagram shows a block of wood in the shape of a cuboid.



The density of the wood is 0.6 g/cm^3 .

Work out the mass of the block of wood.

Volume of block = $l \times w \times h$

$= 12 \times 10 \times 4 = 480 \text{ cm}^3$

Work out the volume of the block in cm^3 .

Density = $\frac{\text{mass}}{\text{volume}}$

$0.6 = \frac{\text{mass}}{480}$

Substitute values into the formula.

$0.6 \times 480 = \frac{\text{mass} \times 480}{480}$

Multiply both sides by 480.

Mass = 288 g

Key Terms:

Speed: Measures how fast an object is going.

Density: Measures how compact an object is.

Pressure: Measures the amount of force exerted by a unit area, on an object.

Multiplier: The number you multiply by to calculate a percentage of an amount.

Interest: Money paid regularly at a particular rate for the use of money lent, or for delaying the repayment of a debt.



Knowledge Organiser: 14 Multiplicative Reasoning

Percentages

Percentage Change

$$\frac{\text{Actual Change}}{\text{Original Amount}} \times 100$$

Calculate the percentage change between 50 and 45.

$$50 - 45 = 5 \rightarrow \frac{5}{50} \times 100 = 10\%$$

Percentage Increase/Decrease

Increase 30 by 10%

$$10\% \text{ of } 30 = 3$$

$$30 + 3 = 33$$

Work out 10%

Add your answer to the original number

Decrease 60 by 30%.

$$30\% \text{ of } 60 = 0.3 \times 60 = 18$$

$$60 - 18 = 42$$

Convert the percentage to a decimal multiplier.

Subtract your answer from the original number.

Compound interest

Compound Interest means that you work out the interest for the first period, add it to the total, and then calculate the interest for the next period etc.

Below is a formula we can use to make the calculation quicker.

$$\text{Amount of money after } x \text{ years} = \text{amount} \times \text{multiplier}^x$$

Tess invested £5000 at 4% compound interest for five years. How much was the investment worth after five years?

$$5000 \times 1.04^5 = £6083.26$$

Remember to round money to 2dp.

$$100\% + 4\% = 104\% = 1.04$$

Reverse percentages

Reverse percentages are used when the percentage and the final number is given, and the original number needs to be found.

Q: A shop offers 30% off in a sale. The sale price of a pair of shoes is £84. Calculate the cost of the shoes before the sale.

$$\text{£84} = 70\% \text{ of the original amount.} \rightarrow \text{A: } \frac{84}{70} \times 100 = \text{£120}$$

$$\frac{\text{Value}}{\text{Equivalent percentage}} \times 100 = \text{Original amount}$$

Q: After a 4% wage rise, Bill earned £1248 each month. Calculate how much Bill earned each month before the wage rise.

$$\text{A: } \frac{1248}{104} \times 100 = \text{£1200}$$

£1248 = 104% of the original amount.

Simple Interest

Find the simple interest when £5000 is invested at 2.75% per annum over 2 years.

$$2.75\% = 0.0275$$

Convert the percentage to a decimal multiplier.

$$5000 \times 0.0275 = \text{£137.50}$$

This is the interest earned over 1 year.

$$\text{£137.50} \times 2 = \text{£275}$$

Multiply your answer by 2.

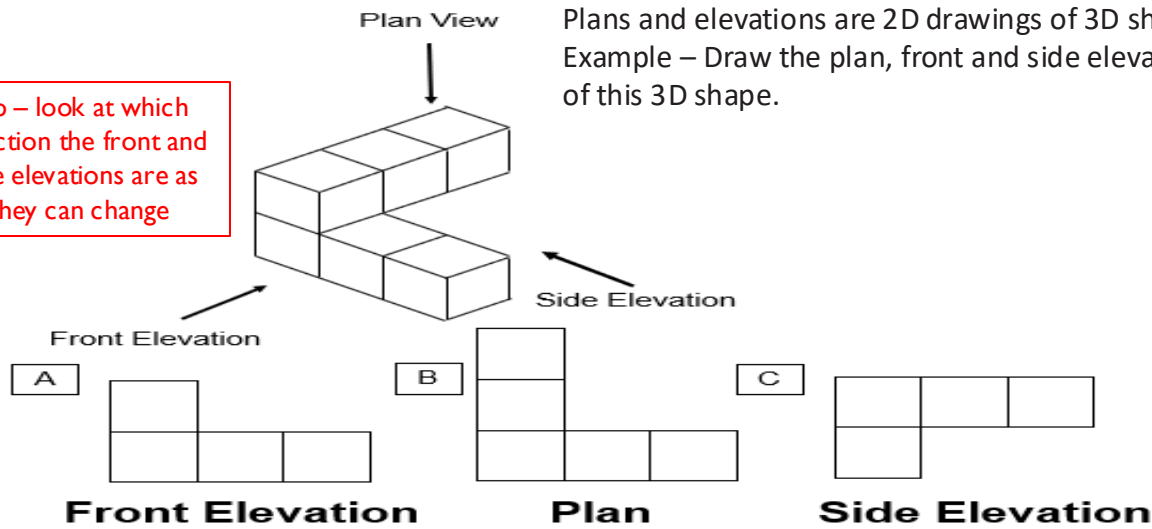


Knowledge Organiser: 15 Plans and Elevations

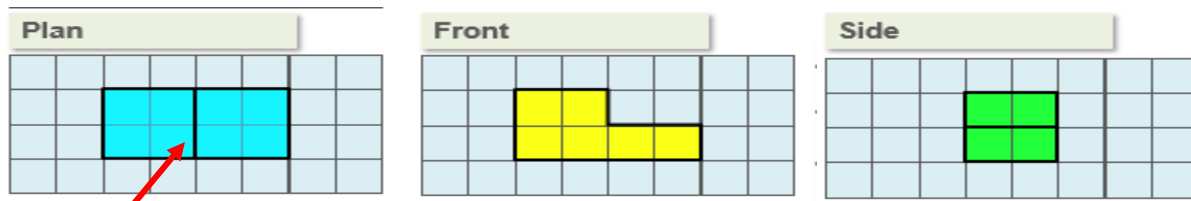
What you need to know:

Drawing plan, front and side elevations

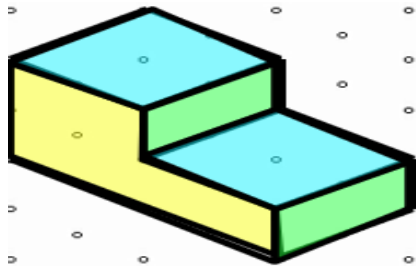
Tip – look at which direction the front and side elevations are as they can change



Example – Draw the 3D shape which has the following plan and elevations.



A solid thick line indicates a change of depth



Key Terms:

Plan – what the shape looks like from above

Front Elevation – this is the view you would see directly in front of you

Side Elevation – what the shape looks like from the side

3D Shape – a three-dimensional shape can be defined as a solid figure or an object or shape that has three dimensions – length, width and height

Face – A face is a 2D shape that makes up one surface of a 3D shape,

Edge – an edge is a line segment where two faces meet.

Vertex – is a point where two or more lines segments or edges meet.

You need to be able to:

- Understand clockwise and anticlockwise
- Draw circles and arcs
- Measure and draw lines and angles
- Know the terms face, edge and vertex.
- Know and use compass directions
- Identify and sketch planes of symmetry of 3D solids
- Understand and draw front and side elevations and plan views of shapes made from simple solids.
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid.
- Be able to sketch a 3D Shape using the dimensions given.

Reminder:

Clockwise direction – the direction the hands of a clock go (right)

Anti-clockwise – the opposite (left)



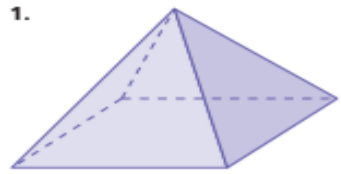
Knowledge Organiser: 15a Plans and Elevations

What you need to know:

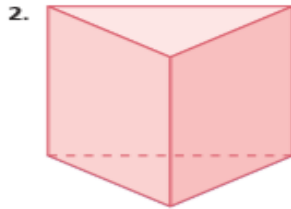
Properties of 3D Shapes

3-dimensional shapes have faces, edges and vertices.

Example – How many faces, edges and vertices do each of these shapes have?



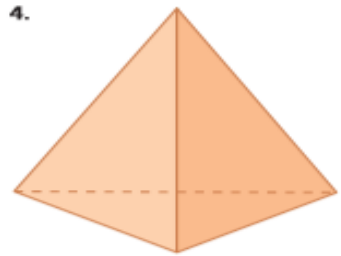
Faces = **5**
Edges = **8**
Vertices = **5**



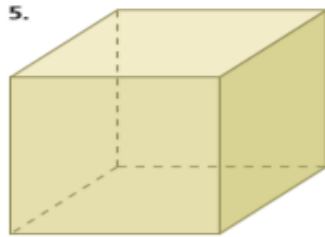
Faces = **5**
Edges = **9**
Vertices = **6**



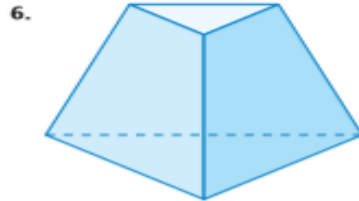
Faces = **6**
Edges = **12**
Vertices = **8**



Faces = **4**
Edges = **6**
Vertices = **4**



Faces = **6**
Edges = **12**
Vertices = **8**

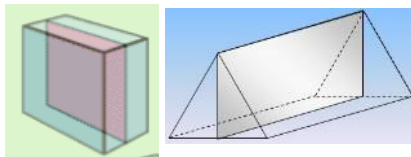


Faces = **5**
Edges = **9**
Vertices = **6**

TIP – don't forget any that you cannot see

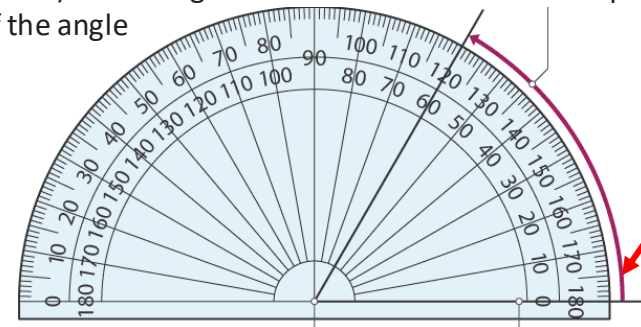
Reminder

Symmetry – when two or more parts are identical after a flip or turn
A **plane of symmetry** bisects a 3D shape into halves that are mirror images of each other. Examples:



Measuring and Drawing Angles

When measuring angles, make sure that the centre of the protractor is over the **vertex** (corner) of the angle and that the base line of the protractor is along one of the lines of the angle



Always read from zero.
In this example use the inside scale

Ensure the centre and base line are lined up with the angle lines

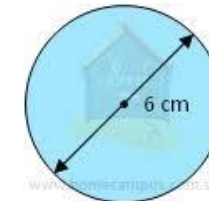
To draw an angle start by drawing a straight line. Line the protractor up and mark the required angle. Join the vertex of your line and mark up.

TIP – when drawing and measuring always estimate the size of the angle first

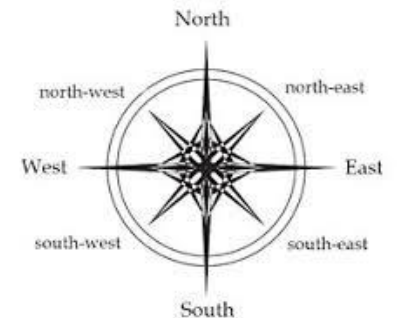
Drawing Circles

Example – draw a circle with diameter 6cm

1. Diameter = 6cm therefore radius – 3cm.
2. Open up your compass and measure 3cm.
3. Draw the circle by turning your compass 360°



Compass Directions





Knowledge Organiser: 15b Loci and Construction

What you need to know:

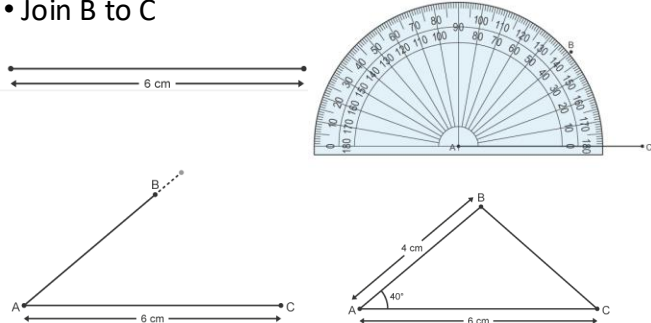
Constructing Triangles

In order to draw a triangle accurately you need either:

- SAS – side, angle side: two sides and the angle in between
- ASA – angle, side, angle: two angles and the side in between
- SSS – side, side, side: the lengths of the three sides

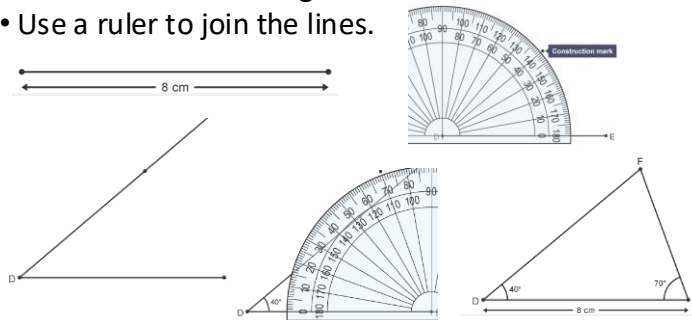
Example – draw an accurate triangle with lengths 4cm and 6cm with an angle of 40° in between.

- Draw the longest side with your ruler
- Using a protractor measure the angle and mark it B
- Use a ruler to measure and draw the line from A to B so that it is the other length
- Join B to C



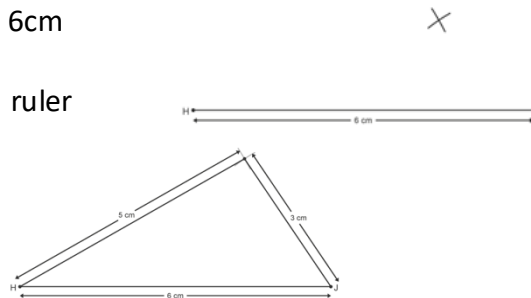
Example – draw an accurate triangle with angles 40° and 70° , with a side of length 8cm in between

- Draw the side using a ruler
- Use a protractor to measure one of the angles – put a construction mark on.
- Use a ruler to draw a line from the end of the line segment through the construction mark
- Put the protractor on the other end of the line and measure the other angle. Put a construction mark on.
- Use a ruler to join the lines.



Example – draw an accurate triangle with sides 3cm, 5cm and 6cm

- Draw the longest side with your ruler
- Open a pair of compasses to one of the other lengths (use a ruler to measure accurately)
- Put the compass on one end of the line and draw an arc
- Open the pair of compasses up to the final length
- Put it on the other end and draw an arc
- Join each end of the line up with where the arcs intersect



Key Terms:

Locus (plural – Loci) – a path formed by a point which moves according to some rule.

Constructions - are accurate diagrams drawn using a pair of compasses and a ruler.

Perpendicular - If the angle between two lines is a right angle, the lines are said to be perpendicular.

Bisect - To divide into two equal sections, cut in half.

Equidistant – equal distance

Region – the area that satisfies a set of instructions

You need to be able to:

- Understand congruence
- Use straight edge and a pair of compasses for standard constructions
- Construct triangles using SAS, ASA or SSS
- Draw and construct diagrams from given specifications
- Find and describe regions satisfying a combination of loci
- Use constructions to solve 2D loci problems

Congruence

You will visit congruency more in unit 19.

If two shapes are **congruent**, they have the **same** area, size, angles and lengths as each other



Knowledge Organiser: 15b Loci and Construction

What you need to know:

How to draw loci and constructions

Equidistant from a point – draw a circle

- Measure the distance required on your ruler
- Place your compass point in the centre
- Draw round 360°

Every point on the circle is the same distance from the centre



Cut an angle exactly in half – Angle Bisector

- Put your compass on the vertex
- Draw an arc
- Lift your compass and put it on where the arc crosses the lines (intersection points)
- Draw an arc from each intersection point
- Draw a straight line from the vertex through where the two new arcs cross



Cut a line exactly in half – Perpendicular Bisector

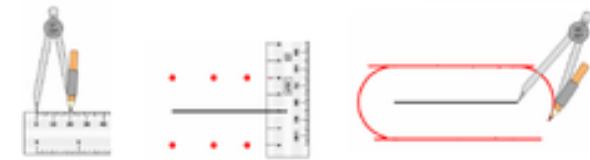
- Put the compass on the end the line segment
- Open the compass to **more than** halfway
- Draw an arc above and below the line
- Put the compass on the other end of the line segment
- Draw an arc above and below the line
- You will see that the arcs intersect
- Draw a line through where both arcs cross.
- You will create a 90° angle.



The perpendicular bisector is used to find:
The midpoint of a line segment
The set of points (a line) equidistant from two points
The region closer to one point than another

Equidistant from a line – draw a 'running track'

- Use a ruler and measure the points parallel to each side of the line segment
- Open your compass at the distance required
- Put your compass on the end of the line segment and draw a semi-circle until it intersects the horizontal lines you have drawn
- Repeat at the other end

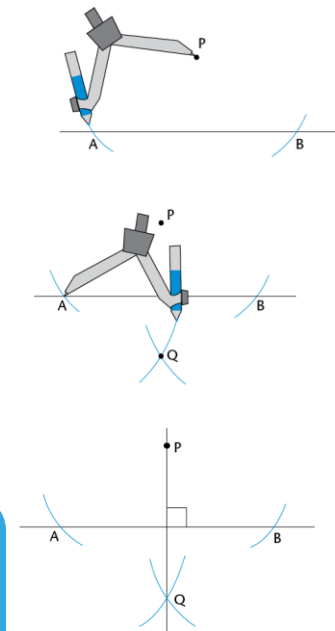


Tip:

Never rub out any construction marks

Draw a perpendicular from a point to a line

- Place your compass on the given point (point P).
- Draw an arc across the line on each side of the given point. Do not adjust the compass width when drawing the second arc.
- From each arc on the line, draw another arc on the opposite side of the line from the given point (P). The two new arcs will intersect.
- Use your ruler to join the given point (P) to the point where the arcs intersect (Q).





Knowledge Organiser: 16 Expand and Simplify Brackets

Key Concepts

Expanding brackets

Single: Where each term inside the bracket is multiplied by the term on the outside of the bracket.

Double: Where each term in the first bracket is multiplied by all terms in the second bracket.

Factorising expressions

Putting an expression back into brackets. To "factorise fully" means take out the HCF.

Difference of two squares

When two brackets are repeated with the exception of a sign change. All numbers in the original expression will be square numbers.

Examples

Linear expressions

Expand and simplify where appropriate

$$1) \quad 7(3 + a) = 21 + 7a$$

$$2) \quad 2(5 + a) + 3(2 + a) = 10 + 2a + 6 + 3a = 5a + 16$$

$$3) \text{ Factorise } 9x + 18 = 9(x + 2)$$

$$4) \text{ Factorise } 6e^2 - 3e = 3e(2e - 1)$$

Quadratic expressions

Expand and simplify:

$$1) \quad (p + 2)(2p - 1) \\ = 2p^2 + 4p - p - 2 \\ = 2p^2 + 3p - 2$$

$$2) \quad (p + 2)^2 \\ (p + 2)(p + 2) \\ = p^2 + 2p + 2p + 4 \\ = p^2 + 4p + 4$$

Factorise:

$$3) \quad x^2 - 2x - 3 \\ = (x - 3)(x + 1)$$

Factorise and solve:

$$4) \quad x^2 + 4x - 5 = 0 \\ (x - 1)(x + 5) = 0 \\ \text{Therefore the solutions are:} \\ \text{Either } x - 1 = 0 \\ \quad \quad \quad x = 1 \\ \text{Or } x + 5 = 0 \\ \quad \quad \quad x = -5$$

Key Words

Expand
Factorise
Simplify
Product
Solve

1) **Expand and simplify** (a) $3(2 - 7f)$

(b) $5(m - 2) + 6$

(c) $3(4 + t) + 2(5 + t)$

2) **Factorise** (a) $6m + 12t$

(b) $9t - 3p$

(c) $4d^2 - 2d$

3) **Expand** $(5g - 4)(2g + 1)$

4) (a) **Factorise** $x^2 - 8x + 15$ (b) **Factorise and solve** $x^2 + 7x + 10 = 0$

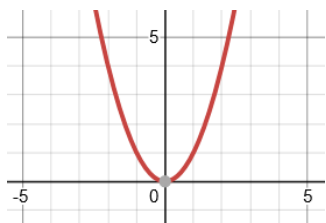


Knowledge Organiser: 16 Quadratic Graphs

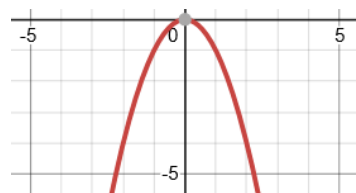
Key Concepts

A quadratic graph will always be in the shape of a parabola.

$$y = x^2$$



$$y = -x^2$$



The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.

Key Words

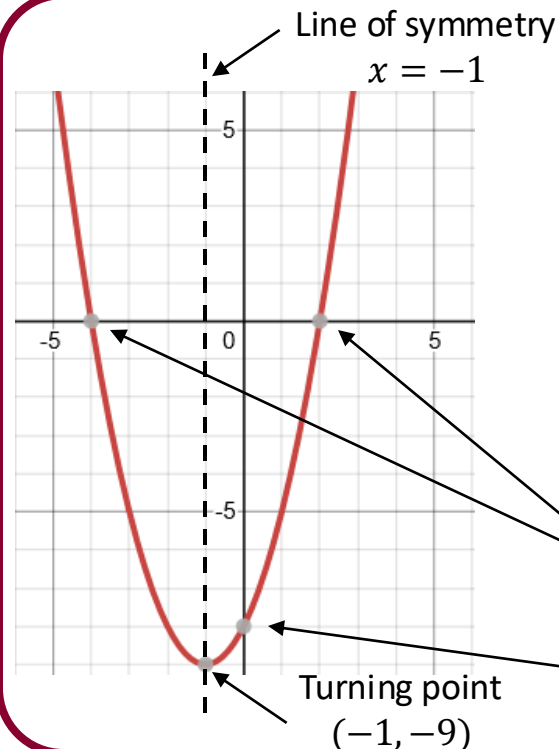
Quadratic
Roots
Intercept
Turning point
Line of symmetry

Examples

$$y = x^2 + 2x - 8$$

A quadratic equation can be solved from its graph.

The roots of the graph tell us the possible solutions for the equation. There can be 1 root, 2 roots or no roots for a quadratic equation. This is dependent on how many times the graph crosses the x axis.



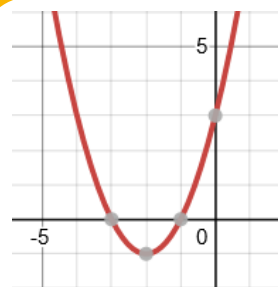
Roots $x = -4$
 $x = 2$

y intercept = -8

Turning point
 $(-1, -9)$

Identify from the graph of $y = x^2 + 4x + 3$:

- 1) The line of symmetry
- 2) The turning point
- 3) The y intercept
- 4) The two roots of the equation



ANSWERS 1) $x = -2$ 2) $(-2, -1)$ 3) 3 4) $x = -1$ and $x = -3$



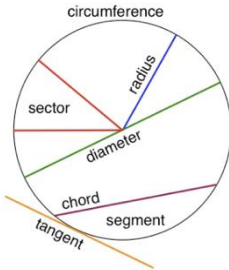
Knowledge Organiser: 17 Perimeter and Circumference

Key Concepts

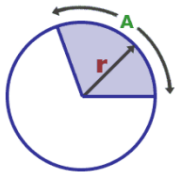
Parts of a circle

Circumference

of a circle is calculated by πd and is the distance around the circle.



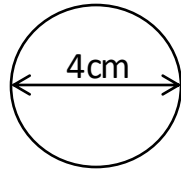
Arc length of a sector is calculated by $\frac{\theta}{360} \pi d$.



Examples

Calculate:

a) Circumference



$$C = \pi \times 4$$

$$= 4\pi$$

$$\text{or } = 12.57\text{cm}$$

b) Diameter when the circumference is 20cm

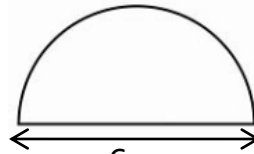
$$C = \pi \times d$$

$$20 = \pi \times d$$

$$\frac{20}{\pi} = d$$

$$\text{Or } 6.37\text{cm}$$

c) Perimeter



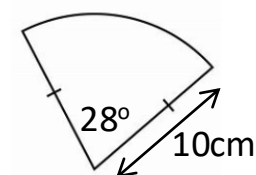
$$P = \frac{\pi \times d}{2} + d$$

$$P = \frac{\pi \times 6}{2} + 6$$

$$P = 3\pi + 6$$

$$\text{Or } = 15.42\text{cm}$$

d) Arc length



$$\text{Arc} = \frac{\theta}{360} \times \pi \times d$$

$$\text{Arc} = \frac{28}{360} \times \pi \times 2 \times 10$$

$$\text{Arc} = \frac{28}{360} \times \pi \times 20$$

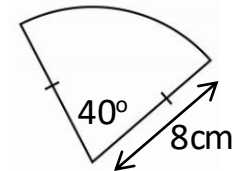
$$\text{Arc} = \frac{14}{9} \pi$$

$$\text{Or } = 4.89\text{cm}$$

Key Words
 Circle
 Perimeter
 Circumference
 Radius
 Diameter
 Pi
 Arc

Calculate:

- 1) The circumference of a circle with a diameter of 12cm
- 2) The diameter of a circle with a circumference of 30cm
- 3) The perimeter of a semicircle with diameter 15cm
- 4) The arc length of the diagram



ANSWERS: 1) 12π or 37.7cm 2) $\frac{\pi}{30}$ or 9.54cm 3) 38.56cm 4) $\frac{9}{16}\pi$ or 5.59cm

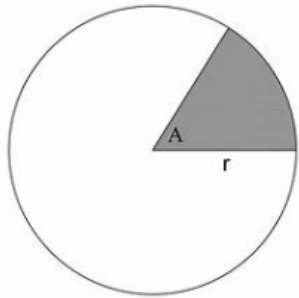


Knowledge Organiser: 17 Area of Circles and Part Circles

Key Concepts

The **area** of a circle is calculated by πr^2

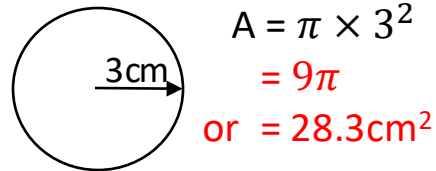
The **area of a sector** is calculated by $\frac{\theta}{360} \pi r^2$



Examples

Calculate:

a) **Area**



$$A = \pi \times 3^2$$

$$= 9\pi$$

$$\text{or } = 28.3\text{cm}^2$$

b) **Radius** when the area is 20cm^2

$$A = \pi \times r^2$$

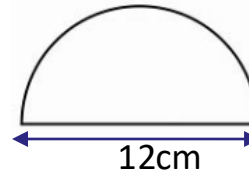
$$20 = \pi \times r^2$$

$$\frac{20}{\pi} = r^2$$

$$\sqrt{\frac{20}{\pi}} = r$$

$$\text{Or } 2.52\text{cm}$$

c) **Area**



$$P = \frac{\pi \times r^2}{2}$$

$$P = \frac{\pi \times 6^2}{2}$$

$$P = 18\pi$$

$$\text{Or } = 56.55\text{cm}^2$$

d) **Area of a sector**

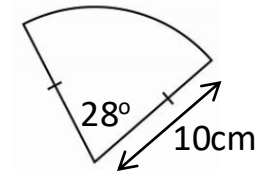
$$\text{Area} = \frac{\theta}{360} \times \pi \times r^2$$

$$\text{Area} = \frac{28}{360} \times \pi \times 10^2$$

$$\text{Area} = \frac{28}{360} \times \pi \times 100$$

$$\text{Area} = \frac{70}{9} \pi$$

$$\text{Or } = 24.43\text{cm}^2$$

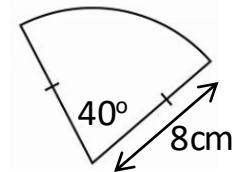


Key Words

Circle
Area
Radius
Diameter
Pi
Sector

Calculate:

- 1) The area of a circle with a radius of 9cm
- 2) The radius of a circle with an area of 45cm^2
- 3) The area of a semicircle with diameter of 16cm
- 4) The area of the sector in the diagram



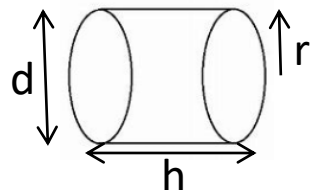
ANSWERS: 1) 81π or 254.47cm^2 2) $\sqrt{\frac{45}{\pi}}$ or 3.78cm 3) 32π or 100.53cm^2 4) $\frac{9}{64}\pi$ or 22.34cm^2



Knowledge Organiser: 17 Volume and Surface Areas of Cylinders

Key Concepts

A **cylinder** is a right **prism** with the cross section of a circle.

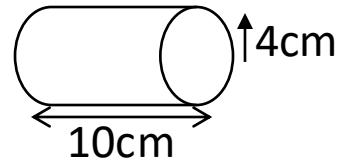


The **volume** of a cylinder is calculated by $\pi r^2 h$ and is the space inside the 3D shape

The **surface area** of a cylinder is calculated by $2\pi r^2 + \pi dh$ and is the total of the areas of all the faces on the shape.

Examples

From the diagram calculate:



a) **Volume**

$$V = \pi \times r^2 \times h$$

$$V = \pi \times 4^2 \times 10$$

$$V = 160\pi$$

$$\text{Or } = 502.65\text{cm}^3$$

b) **Surface Area** – You can use the net of the shape to help you

Area of two circles

$$= 2 \times \pi \times r^2$$

$$= 2 \times \pi \times 4^2$$

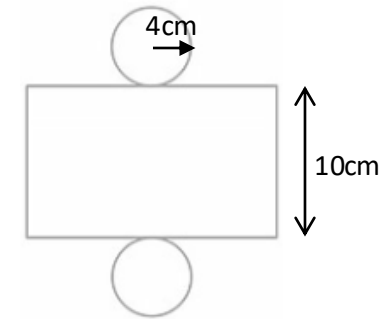
$$= 32\pi$$

Area of rectangle

$$= \pi \times d \times h$$

$$= \pi \times 8 \times 10$$

$$= 80\pi$$



$$\text{Surface Area} = 32\pi + 80\pi$$

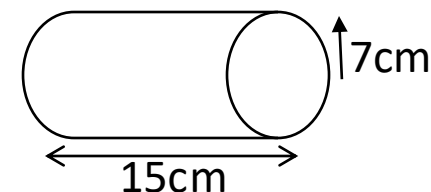
$$= 112\pi$$

$$\text{or } = 351.86\text{cm}^2$$

Key Words

Cylinder
Surface Area
Radius
Diameter
Pi
Volume
Prism

Calculate the volume and surface area of this cylinder





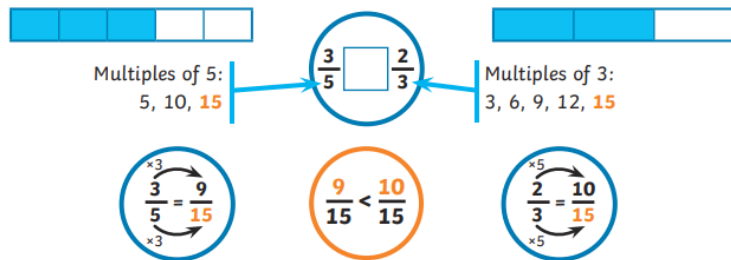
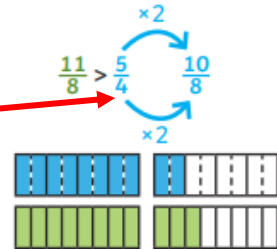
Knowledge Organiser: 18a Fractions

What you need to know:

Comparing and ordering fractions

Comparing fractions: To compare two fractions you need to make the denominators the same.

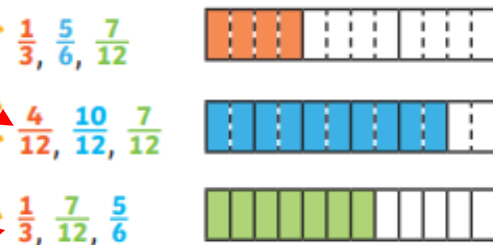
The second fraction can be changed into eighths by multiplying by 2.



Ordering fractions: To compare a list of fractions you need to make the denominators the same. Below all of the denominators have been turned into 12 because 3, 6 and 12 are all factors of 12.

Making the denominators the same is important. Here we multiplied the red fraction by 4 and the blue by 2.

We then order from smallest to biggest writing them as the original fractions.



Key Terms:

Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).

Integer: Whole number.

Simplify: Divide the numerator and denominator by a common factor.

Ascending Order: Place in order, smallest to largest.

Descending Order: Place in order, largest to smallest.

Equivalent: Of equal value.

Mixed number: A whole number with a fraction.

Improper fraction: A fraction where the numerator is bigger than the denominator.

Reciprocal: (multiplicative inverse) Turn the fraction upside down. One of a pair of numbers that when multiplied together equal 1.

You need to be able to:

- Add and subtract fractions with different denominators.
- Multiply and divide fractions.
- Complete calculations with mixed numbers.
- Simplify a fraction and identify equivalent fractions.
- Calculate a fraction of an amount.
- Write the reciprocal of a number.



Knowledge Organiser: 18a Fractions

What you need to know:

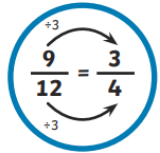


Factors of 9:

1, 3, 9

Factors of 12:

1, 2, 3, 4, 6, 12



Simplifying fractions

Simplifying: You need to identify factors of the numerator and denominator. Once the factors have been identified you need to find the highest number that appears in both lists, this is the number you will divide your fraction by. Divide the numerator and denominator by this divisor separately to get your answer.

Fractions of amounts

We need to divide by the denominator to calculate one part and then multiply by the numerator to work out x parts.

Calculate $\frac{2}{3}$ of 18.



$\div 3$

$18 \div 3 = 6$ this is equal to $\frac{1}{3}$

$2 \times 6 = 12$ this is equal to $\frac{2}{3}$

Reciprocals

To get the reciprocal of a number we divide 1 by that number.

Examples:

Reciprocal of 8 is $\frac{1}{8}$

Reciprocal of 5 is $\frac{1}{5}$

Reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

Adding and subtracting fractions

To add and subtract fractions you must make the denominator the same.

$$\frac{1}{2} + \frac{2}{3} = \frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$$

Cross multiply to get your new numerators. Pink multiplied by pink, blue by blue.

Multiply the 2 denominators to get a common denominator.

Add the numerators together then convert into a mixed number.

The method for subtracting fractions is exactly the same, we just subtract our two fractions at the last stage instead of adding them.

Multiplying and dividing fractions

When multiplying and dividing fractions the denominator does not have to be the same.

$$\frac{1}{2} \times \frac{2}{3} = \frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$$

Multiply the numerators together and then multiply the denominators.

$$\frac{2}{5} \times 3 = \frac{2}{5} \times \frac{3}{1} = \frac{2}{5} \times \frac{3}{1} = \frac{6}{5} = 1\frac{1}{5}$$

Change the whole number to a fraction and then multiply.

To divide fractions we use KFC, keep flip change.

$$\frac{3}{9} \div \frac{1}{2} = \frac{3}{9} \times \frac{2}{1} = \frac{6}{9}$$

Keep the same

Change from $\div$ to $\times$

Flip over

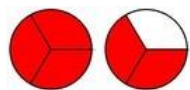


Knowledge Organiser: 18a Fractions

What you need to know:

Mixed numbers and improper fractions

Mixed numbers: This is when you have a whole number and a fraction. When calculating with mixed numbers we need to be able to change them into improper fractions first.



$$1\frac{2}{3} = \frac{1 \times 3 + 2}{3} = \frac{5}{3}$$

The denominator does not change.

Multiply the whole number by the denominator and add the numerator of the fraction.

Improper fractions: This is when the numerator is bigger than the denominator. It can be changed into a mixed number.

The denominator does not change.

$$\frac{13}{5}$$

There are 2 fives in 13 so the whole number is 2.

$$2\frac{3}{5}$$

The remainder is 3 so there are 3 fifths left.

Simplifying mixed numbers

Simplifying: We need to check that the fraction or mixed number is in the smallest numbers possible.

20 and 24 are both in the 4 times table so we must divide both by 4 to simplify.

$$5\frac{20}{24} = 5\frac{5}{6}$$

Change to a mixed number then divide 2 and 4 by 2 to simplify it.

$$\frac{18}{4} = 4\frac{2}{4} = 4\frac{1}{2}$$

Adding and subtracting mixed numbers

Adding mixed numbers: We need to change them into improper fractions and then make the denominators the same.

Change the mixed numbers to improper fractions.

$$1\frac{1}{2} + 2\frac{1}{3}$$

Multiply the 2 denominators to get a common denominator.

$$= \frac{3}{2} + \frac{7}{3}$$

Add the numerators together then convert into a mixed number.

$$= \frac{9}{6} + \frac{14}{6} = \frac{23}{6} = 3\frac{5}{6}$$

Subtracting mixed numbers The method for subtracting fractions is exactly the same, we just subtract our two fractions at the last stage instead of adding them.

Change the mixed numbers to improper fractions.

$$4\frac{1}{2} - 2\frac{1}{3}$$

Multiply the 2 denominators to get a common denominator.

$$= \frac{9}{2} - \frac{7}{3}$$

Subtract the numerators then convert into a mixed number.

$$= \frac{27}{6} - \frac{14}{6} = \frac{13}{6} = 2\frac{1}{6}$$



Knowledge Organiser: 18b Indices and Standard Form

What you need to know:

Laws of indices

Multiplication law: When multiplying with the same base (number/letter) we add the powers.

$$\text{General rule: } a^m \times a^n = a^{m+n}$$

$$2^5 \times 2^7 = 2^{5+7} = 2^{12}$$

$$x^3 \times x^8 = x^{3+8} = x^{11}$$

When multiplying the terms we add the powers together.

Division law: When dividing with the same base (number/letter) we subtract the powers.

$$\text{General rule: } a^m \div a^n = a^{m-n}$$

$$2^{14} \div 2^7 = 2^{14-7} = 2^7$$

$$x^{10} \div x^8 = x^{10-8} = x^2$$

When dividing the terms we subtract the powers together.

Brackets law: When raising a power to another power we multiply the powers together.

$$\text{General rule: } (a^m)^n = a^{m \times n}$$

$$(5^4)^2 = 5^{4 \times 2} = 5^8$$

$$(h^9)^3 = h^{9 \times 3} = h^{27}$$

When raising to a power we multiply the powers together.

Key facts: You need to also remember that:

$$p = p^1$$

$$p^0 = 1$$

Anything to the power zero is equal to 1.

Key Terms:

Square: A square number is the result of multiplying a number by itself.

Cube: A cube number is the result of multiplying a number by itself twice.

Root: A root is the inverse of a power.

Prime number: A prime is a number that has exactly two distinct factors, which are 1 and itself.

Indices: These are the squares, cubes and powers.

Standard form: This is a way of writing really big or really small numbers.

Ordinary numbers: These are numbers that are not yet in standard form. They are the normal form we see numbers in.

You need to be able to:

- Use the laws of indices to simplify expressions with numbers and in algebraic form.
- Convert between ordinary numbers and standard form and vice versa.
- Multiply and divide with standard form.
- Add and subtract with standard form.



Knowledge Organiser: 18b Indices and Standard Form

What you need to know:

Converting with standard form

Ordinary numbers: To change between ordinary numbers and standard form we need to use a power of 10.

$$120000 = 1.2 \times 10^5$$

$$0.005 = 5 \times 10^{-3}$$

This number needs to be bigger than or equal to 1 and less than 10 to be in standard form.

Positive power = very big number.
Negative power = very small number.

Standard form: To change numbers from standard form back to ordinary numbers we multiply by the power of 10.

$$7.32 \times 10^4 = 73200$$

$$2.4 \times 10^{-3} = 0.0024$$

The power tells us how many places to move the "." not how many zeros to add.

Multiplying standard form

Multiply standard form: We multiply the numbers and add the powers.

$$(5 \times 10^4) \times (7 \times 10^6)$$

$$= 35 \times 10^{10}$$

$$= 3.5 \times 10^{11}$$

This is not in standard form because 35 is not less than 10.

$$(3.2 \times 10^3) \times (4 \times 10^4)$$

$$= 12.8 \times 10^7$$

$$= 1.28 \times 10^8$$

Remember to add the powers together.

Dividing standard form

Divide standard form: We divide the numbers and subtract the powers.

$$(8 \times 10^9) \div (2 \times 10^6)$$

$$= 4 \times 10^3$$

This is already in standard form because 4 is less than 10.

$$(1.2 \times 10^5) \div (2 \times 10^2)$$

$$= 0.6 \times 10^3$$

$$= 6 \times 10^2$$

This is not in standard form because 0.6 is less than 1.

Adding and subtracting standard form

To add and subtract with standard form we must convert out of standard form into ordinary numbers first and then add/subtract.

$$(8.1 \times 10^5) + (2 \times 10^3)$$

$$= 810000 + 2000$$

$$= 812000$$

$$= 8.12 \times 10^5$$

Convert into ordinary numbers.

Add/subtract.

Change back into standard form.

$$(2.35 \times 10^5) - (4.1 \times 10^3)$$

$$= 235000 - 4100$$

$$= 230900$$

$$= 2.309 \times 10^5$$

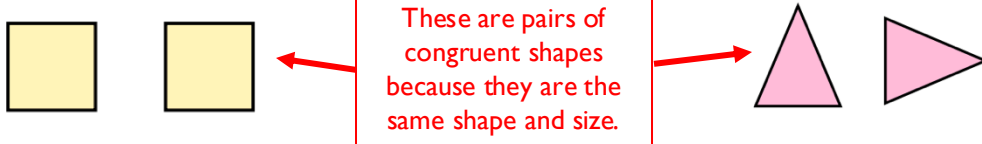


Knowledge Organiser: 19a Similarity and Congruence

What you need to know:

Congruent and Similar Shapes

Congruent: This is when the shapes are an identical shape and size.



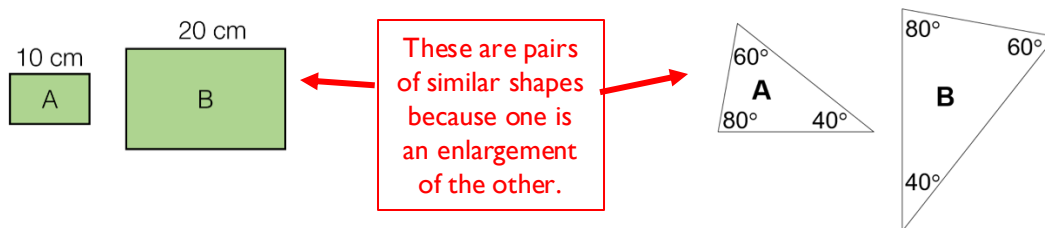
Checking Congruence in Triangles:

SSS – two triangles with all three sides equal

SAS – two triangles where we know two sides and the included angle are equal

ASA – two triangles where we know two angles and the include side are equal

Similar: This is when the shapes have the same size angles, or one is an enlargement of the other.



Key Terms:

Congruent: These shapes are the same shape and same size but can be in any orientation.

Similar: Two shapes are mathematically similar if one is an enlargement of the other.

Scale factor: This is how many times bigger/smaller one shape is in comparison to the other.

You need to be able to:

- Identify the difference between congruent shapes and similar shapes.
- Calculate missing lengths in similar shapes.
- Calculate missing lengths in similar 3D shapes.
- Calculate area and volume of similar 2D and 3D shapes.

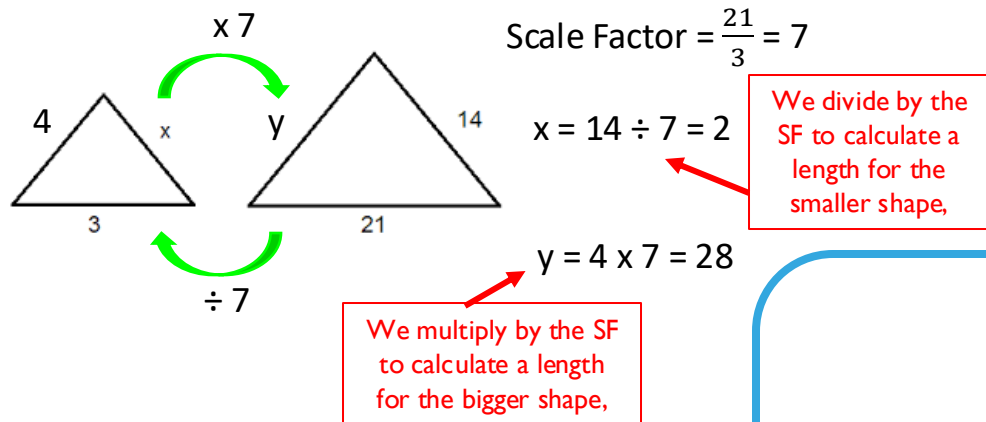


Knowledge Organiser: 19a Similarity and Congruence

What you need to know:

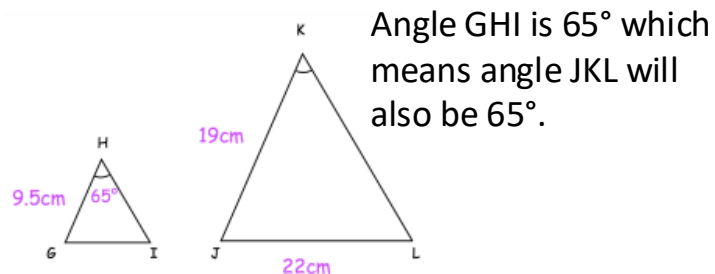
Similar shapes - sides

When we are working with similar shapes, we always need to find the scale factor. This is how many times bigger one shape is than the other.



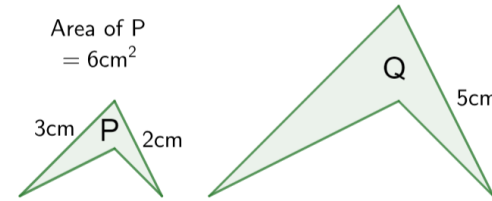
Similar shapes - angles

The angles in similar shape do not change size.



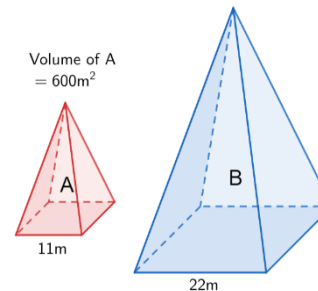
Similar shapes and solids

Area: When calculating with area the scale factor is squared.



We would need to divide if we were calculating for the smaller shape just as we would with lengths.

Volume: When calculating with volume the scale factor is cubed.





Knowledge Organiser: 20 Rearrange and Solve Equations

Key Concepts

Solving equations:

Working with inverse operations to find the value of a variable.

Rearranging an equation:

Working with inverse operations to isolate a highlighted variable.

In solving and rearranging we **undo the operations** starting from the last one.

Examples

Solve:

$$\begin{array}{rcl}
 7p - 5 & = & 3p + 3 \\
 -3p & & -3p \\
 4p - 5 & = & 3 \\
 +5 & & +5 \\
 4p & = & 8 \\
 \div 2 & & \div 2 \\
 p & = & 2
 \end{array}$$

Solve:

$$\begin{array}{rcl}
 5(x - 3) & = & 4(x + 2) \\
 \text{expand} & & \text{expand} \\
 5x - 15 & = & 4x + 8 \\
 -4x & & -4x \\
 x - 15 & = & 8 \\
 +15 & & +15 \\
 x & = & 23
 \end{array}$$

Rearrange to make r the subject of the formulae :

$$\begin{array}{rcl}
 Q & = & \frac{2r - 7}{3} \\
 \times 3 & & \\
 3Q & = & 2r - 7 \\
 +7 & & +7 \\
 3Q + 7 & = & 2r \\
 \div 2 & & \div 2 \\
 \frac{3Q + 7}{2} & = & r
 \end{array}$$

Rearrange to make c the subject of the formulae :

$$\begin{array}{rcl}
 2(3a - c) & = & 5c + 1 \\
 \text{expand} & & \\
 6a - 2c & = & 5c + 1 \\
 +2c & & +2c \\
 6a & = & 7c + 1 \\
 -1 & & -1 \\
 6a - 1 & = & 7c \\
 \div 7 & & \div 7 \\
 \frac{6a - 1}{7} & = & c
 \end{array}$$

Key Words

Solve
Rearrange
Term
Inverse

- 1) Solve $7(x + 2) = 5(x + 4)$
- 2) Solve $4(2 - x) = 5(x - 2)$
- 3) Rearrange to make m the subject $2(2p + m) = 3 - 5m$
- 4) Rearrange to make x the subject $5(x - 3) = y(4 - 3x)$

ANSWERS: 1) $x = 3$ 2) $x = 2$ 3) $m = \frac{3 - 4p}{7}$ 4) $x = \frac{4y + 15}{5 + 3y}$



Knowledge Organiser: 20 Simultaneous Equations

Key Concepts

Simultaneous equations are when **more than one equation** are given, which involve **more than one variable**. The variables have the **same value** in each equation.

Two linear equations:

$$\begin{array}{r} 3x + 2y = 18 \\ 3x - y = 9 \quad \times 2 \end{array}$$

$$\begin{array}{r} 3x + 2y = 18 \\ 6x - 2y = 18 \quad + \\ \hline 9x = 36 \\ x = 4 \end{array}$$

Substitute in $x = 4$ into an original equation

$$\begin{array}{l} 3x + 2y = 18 \\ (3 \times 4) + 2y = 18 \\ 12 + 2y = 18 \\ 2y = 6 \\ y = 3 \end{array}$$

Examples

SSS – Same Sign Subtract
DSA – Different Sign Add

$$5x + 2y = 9$$

$$10x + 3y = 16$$

Multiply the first equation by 2.

$$10x + 4y = 18$$

$$10x + 3y = 16$$

Same Sign Subtract (+10x on both)
 $y = 2$

Substitute $y = 2$ in to equation.

$$5x + 2 \times 2 = 9$$

$$5x + 4 = 9$$

$$5x = 5$$

$$x = 1$$

Solution: $x = 1, y = 2$

Key Words

Simultaneous
Substitution
Elimination
Linear
Quadratic

Solve each set of simultaneous equations:

$$\begin{array}{l} 1) \quad 3x + 2y = 4 \\ \quad \quad 4x + 5y = 17 \end{array}$$

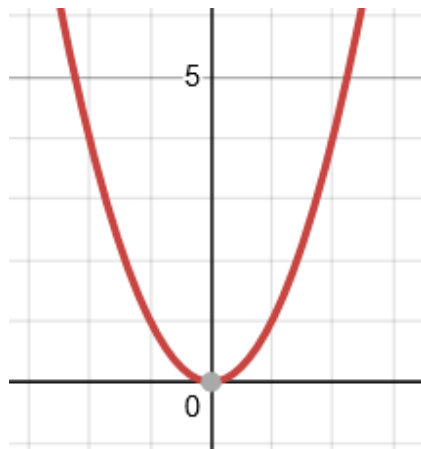
$$\begin{array}{l} 2) \quad 2x + 4y = 26 \\ \quad \quad 3x - y = 4 \end{array}$$

ANSWERS: 1) $x = -2$ and $y = 5$ 2) $x = 3$ and $y = 5$

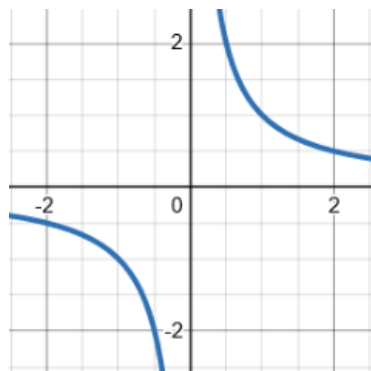


Knowledge Organiser: 20 Types of Graph

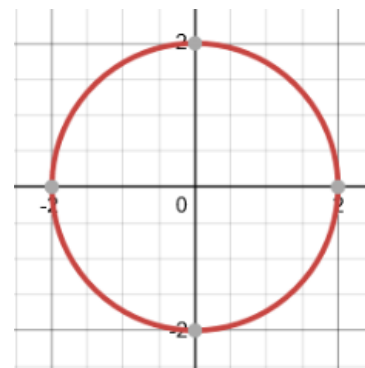
Examples



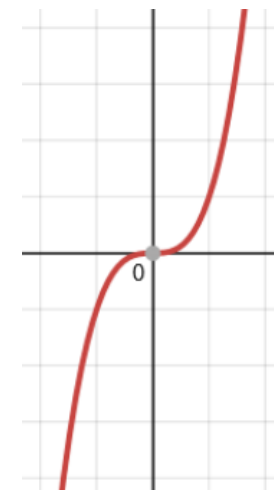
Quadratic graphs
 $y = x^2$



Reciprocal graphs
 $y = \frac{1}{x}$



Circle graphs
 $x^2 + y^2 = 4$

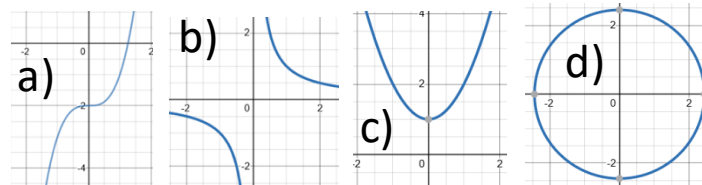


Cubic graphs
 $y = x^3$

Key Words

Quadratic
Cubic
Reciprocal
Circle
Graph

Match the graph with the correct equation:



- 1) $x^2 + y^2 = 6$
- 2) $y = \frac{1}{x}$
- 3) $y = x^3 - 2$
- 4) $y = x^2 + 1$