KS4 Maths
Higher
Knowledge Organisers

Knowledge Organiser: 1a Calculations, Checking and Rounding

What you need to know:

How to add, subtract, multiply and divide integers and decimals.

When adding and subtracting important to line up decimal points, then add or subtract as normal.

To multiply with decimal numbers multiply each number by a power of ten until it is an integer. Multiply the numbers together, then divide the answer by the total power of ten.

2.3×0.76

$\downarrow \times 10 \quad \downarrow \times 100$

$23 \times 76 = 1748 \xrightarrow[\div 1000]{} 1.748$

$\times 10 \times 100 = \times 1000$

When dividing with decimals, write the question as a fraction. Multiply the numerator and denominator by the same power of 10 until the denominator is an integer then divide the two numbers.

$$\frac{6.46}{0.8} \overset{\times 10}{\underset{\times 10}{=}} \frac{64.6}{8} = 8.075$$

	0	8.	0	7	5
8	6	4.	6	0	0

How to find the solution to one calculation using another.

Place value can be used to calculate the solution to one calculation when given another

$2.6 \times 15.8 = 41.08$

$\downarrow \times 10 \quad \downarrow \div 100 \quad \downarrow$

$\times 10 \div 100 = \div 10$

$26 \times 0.158 = 4.108$

How to use the product rule for counting.

If there are **m** ways of doing one task and for each of these, there are **n** ways of doing another task, then the total number of ways the two tasks can be done is **m × n** ways.
If a task is to be repeated twice and there are m ways of doing the task, and **the order is important**, the total number of permutations is **m x (m-1).**

Eg. 35 people enter a raffle. How many different ways can the winners of first and second prizes be chosen?

$35 \times 34 = 1190$ ways.

If the same set of objects is used twice and **order is unimportant**, then the total number of combinations is $\frac{m \times (m-1)}{2}$.

Eg. How many ways are there of choosing 2 students from a group of 25.

$$\frac{25 \times 24}{2} = 300 \text{ ways.}$$

Key Terms:

Integer– A whole number

Product rule– A way of finding the total number of outcomes for two or more events.

Permutations– The number of different arrangements when order matters.

Combinations– The number of different arrangements when the order doesn't matter.

Knowledge Organiser: 1a Calculations, Checking and Rounding

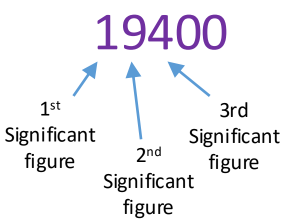
What you need to know:

How to round numbers to a given degree of accuracy.

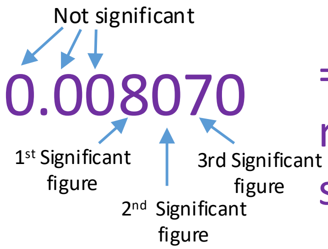
Rounding makes a number simpler but keeps its value close to what it was. If the digit to the right of the rounding digit is less than 5 round down, if it is 5 or more, round up.

Significant figures.

Significant figures (s.f.) start from the first non-zero digit in a number. After that any number (including 0) is the next significant digit.



= 19000 when rounded to 2 significant figures.



= 0.0081 when rounded to 2 significant figures.

How to estimate the solutions to complex calculations by rounding.

To estimate you should round each number in a calculation to 1 significant figure, then calculate. For very large or very small numbers it may be more appropriate to round to 2 or more s.f.

$$\frac{348+692}{0.526} \approx \frac{300+700}{0.5} = 2000$$

How to decide if your solution is an underestimate or overestimate.

Decide if you have made each number bigger or smaller by rounding. When dividing remember that if you divide by a number that has been rounded up, your answer will be an underestimate and vice versa.

Key Terms:

Rounding– Making a number simpler whilst keeping its value close to the original.

Decimal places– The number of digits after the decimal point in a number.

Significant Figures– The number of digits in a value that carry a meaning to the size of the number.

Estimate– Find a value that is close to the right answer by rounding.

≈- Approximately equal to

Overestimate– An estimated value that is higher than the exact value.

Underestimate– An estimated value that is lower than the exact value.

Key Facts:

Money Calculations

When answering questions linked to money, always round your answer to 2 d.p.

Rounding in the real world

Journalists use rounded numbers in headlines to make them easier to understand and have more impact.


DAILY NEWS

FREE GLASTO GUIDE INSIDE!

29 642 tickets sold for tribute band concert!


DAILY NEWS

FREE GLASTO GUIDE INSIDE!

30 000 tickets sold for tribute band concert!

Checking your answers

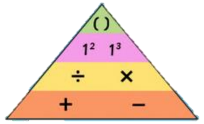
It is useful to estimate complex calculations to help you to check your answers are reasonable – you may even use estimation to check you have enough money when shopping for example.

Knowledge Organiser: 1b Indices, Roots, Reciprocals and Hierarchy of Operations

What you need to know:

How to use the hierarchy of operations

When answering complex questions there is an order in which calculations need to take place.



First answer anything in brackets.
Then calculate any powers or roots.
Next, multiplication and division in order from left to right.
Finally, addition and subtraction in order from left to right.

$$\underbrace{5 \times 4} - \underbrace{8 \div 2} + 3$$

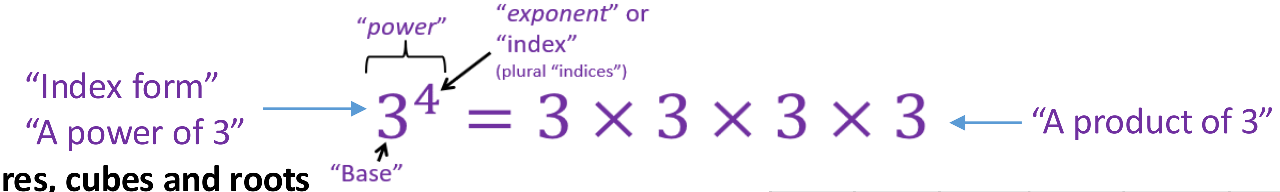
$$\underbrace{20 \quad - \quad 4} \quad + 3$$

$$16 \qquad + 3 = 19$$

$$(2^2 + 6)^2 \times 4 - 8$$
$$\downarrow$$
$$(4 + 6)^2 \times 4 - 8$$
$$\downarrow$$
$$(10)^2 \times 4 - 8$$
$$\downarrow$$
$$100 \times 4 - 8$$
$$\downarrow$$
$$400 - 8 = \mathbf{392}$$

Using index notation (powers)

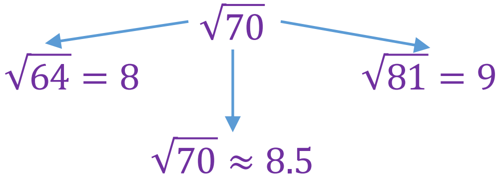
Index notation is a way of writing numbers or letters that have been multiplied by themselves a number of times. The exponent tells us how many times the base appears in a product.



$$3^4 = 3 \times 3 \times 3 \times 3$$

Squares, cubes and roots

The square root $\sqrt{\ }$ is the inverse of squaring a number. The cube root $\sqrt[3]{\ }$ is the inverse of a cube number. To estimate the value of a square root consider which square numbers it is between.



$$\sqrt{70}$$

$$\sqrt{64} = 8 \qquad \sqrt{81} = 9$$

$$\sqrt{70} \approx 8.5$$

1^2	1	6^2	36	11^2	121
2^2	4	7^2	49	12^2	144
3^2	9	8^2	64	13^2	169
4^2	16	9^2	81	14^2	196
5^2	25	10^2	100	15^2	225

1^3	2^3	3^3	4^3	5^3	10^3
1	8	27	64	125	1000

Key Terms:

Index notation – A way of writing numbers or letters that have been multiplied by themselves a number of times.

Square number – The product of a number multiplied by itself.

Cube number– The product of a number multiplied by itself twice, e.g. 5 x 5 x 5 = 125.

Root– The inverse of a square number is a square root. The inverse of a cube number is a cube root

Reciprocal– 1 divided by the number. Also known as the "multiplicative inverse" as if you multiply a number by its reciprocal you always get the answer of 1.

Evaluate– "Find the value of.." give your answer as a number.

Key Facts:

Index Laws – You can only use index laws when the base number is the same.

a^m x $b^n \neq ab^{m+n}$

Mixed products– Multiplication is cumulative – that means you can do it in any order.

So: $2 \times 2 \times 5 \times 2 \times 5 = 2^3 \times 5^2$

Knowledge Organiser: 1b Indices, Roots, Reciprocals and Hierarchy of Operations

What you need to know:

Index laws

Multiplication law: When multiplying with the same base (number/letter) we add the powers. The base stays the same.

General rule: $a^m \times a^n = a^{m+n}$

$2^5 \times 2^7 = 2^{5+7} = 2^{12}$

$x^3 \times x^8 = x^{3+8} = x^{11}$

When multiplying the terms we add the powers together.

Division law: When dividing with the same base (number/letter) we subtract the powers. The base stays the same.

General rule: $a^m \div a^n = a^{m-n}$

$2^{14} \div 2^7 = 2^{14-7} = 2^7$

$x^{10} \div x^8 = x^{10-8} = x^2$

When dividing the terms we subtract the powers together.

Brackets law: When raising a power to another power we multiply the powers together. The base stays the same.

General rule: $(a^m)^n = a^{m \times n}$

$(5^4)^2 = 5^{4 \times 2} = 5^8$

$(4h^9)^3 = 4^3 \times h^{9 \times 3} = 64h^{27}$

When raising to a power we multiply the powers together.

Key facts: You need to also remember that:

$p = p^1$

$p^0 = 1$

Anything to the power zero is equal to 1.

Negative indices: A negative power performs the reciprocal.

General rule: $a^{-m} = \frac{1}{a^m}$

$3^{-1} = \frac{1}{3}$

$\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$

$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Fractional indices: The denominator of a fractional power acts as a root. The numerator of a fractional power acts as a normal power.

General rule: $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

$27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$

$\left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\frac{\sqrt{25}}{\sqrt{16}}\right)^3 = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$

Changing bases: Index laws only work when the base numbers are the same, so sometimes it is necessary to change the base numbers using our knowledge of square and cube numbers.

$Write\ (4)^3\ as\ a\ power\ of\ 2$

$$4 = 2^2\ so\ (4)^3 = (2^2)^3 = 2^6$$

$Write\ (3)^5\ as\ a\ power\ of\ 9$

$$3 = \sqrt{9} = 9^{\frac{1}{2}}$$

$$so\ (3)^5 = \left(9^{\frac{1}{2}}\right)^5 = 9^{\frac{5}{2}}$$

Knowledge Organiser: 1c Factors, Multiples, Primes, Standard Form and Surds

What you need to know:

Finding the Highest Common Factor (Listing)

List the factors of each number and find the highest match.

HCF of 8 and 20

Factors of 8: 1, 2, 4, 8

Factors of 20: 1, 2, 4, 5, 10, 20.

Common factors: 1, 2, 4. so HCF is 4.

Finding the Lowest Common Multiple (Listing)

List multiples of each number until you find a match.

LCM of 3 and 5

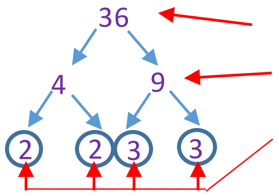
Multiples of 3: 3, 6, 9, 12, 15, 18

Multiples of 5: 5, 10, 15

LCM of 3 and 5 is 15..

Prime factor decomposition

Any number can be written as a product of its prime factors. We can break down or "decompose" a number into its prime factors by using a prime factor tree.



Start with your number

Find any two factors of your starting number. If these numbers aren't prime find factors of these numbers.

Once you get factors are prime circle them and stop. The ends of each "branch" need to all be prime numbers.

Therefore, $36 = 2 \times 2 \times 3 \times 3$

Using index notation: $36 = 2^2 \times 3^2$

Note: this is a **product** of prime factors. You must include the x sign.

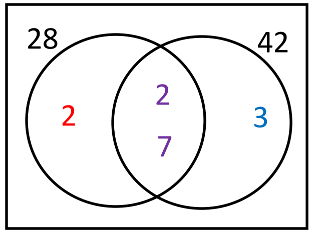
Using Venn Diagrams to find the HCF and LCM

Find the HCF and LCM of 28 and 42

1. Write each number as a product of their prime factors. Use a prime factor tree and don't use index notation.
2. Draw a Venn diagram and label it correctly using 28 and 42.
3. They each have a prime factor of 2 and 7, they will both go in the intersection.
4. The only remaining prime factor for 28 goes on the left, and the remaining prime factor for 42 goes on the right.

$28 = 2 \times 2 \times 7$

$42 = 2 \times 3 \times 7$



The HCF of the two numbers is the product of the two numbers in the intersection.

HCF of 28 and 42 is 7 x 2 = 14

The LCM of the two numbers is the product of all the numbers in the Venn diagram.

LCM of 28 and 42 is 2 x 2 x 7 x 3 = 84

If you are given the HCF and LCM of two numbers and need to find those numbers, you can use a Venn diagram to work backwards.

Key Terms:

Factor – A number that divides exactly into another number without a remainder.

Multiple – The result of multiplying a number by an integer.

Prime number – A number with exactly two factors (itself and 1)

Highest Common Factor (HCF) – The greatest number that divides exactly into two or more numbers.

Lowest Common Multiple (LCM) – The smallest number that is in the times tables of each of the numbers given.

Prime Factor – A factor which is a prime number.

Standard form – A number written in the form $A \times 10^n$ *where* $1 \leq A < 10$ and n is an integer. Also known as Scientific Notation.

Surd – A number written exactly using square or cube roots.

Key Facts:

Factors – it can help to write factors in pairs.

Factors of 12: 1 and 12
2 and 6
3 and 4

Multiples – numbers in the times table

The first 5 multiples of 6 are: 6, 12, 18, 24, and 30.

Prime numbers – you need to know the prime numbers up to 100.

Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Knowledge Organiser: 1c Factors, Multiples, Primes, Standard Form and Surds

What you need to know:

Converting with standard form

Ordinary numbers: To change between ordinary numbers and standard form we need to use a power of 10.

$120000 = 1.2 \times 10^5$

This number need to be bigger than or equal to 1 and less than 10 to be in standard form.

$0.005 = 5 \times 10^{-3}$

Positive power = very big number.
Negative power = very small number.

Standard form: To change numbers from standard form back to ordinary numbers we multiply by the power of 10.

$7.32 \times 10^4 = 73200$

$2.4 \times 10^{-3} = 0.0024$

The power tells us how many places to move not how many zeros to add.

You can use this button on your calculator to write numbers in standard form.

For 2×10^5 you would type

Multiplying standard form

Multiply standard form: We multiply the numbers and add the indices.

$(5 \times 10^4) \times (7 \times 10^6)$

$= 35 \times 10^{10}$

$= 3.5 \times 10^{11}$

This is not in standard form because 35 is not less than 10.

$(3.2 \times 10^3) \times (4 \times 10^4)$

$= 12.8 \times 10^7$

$= 1.28 \times 10^8$

Remember to add the powers together.

Dividing standard form

Divide standard form: We divide the numbers and subtract the indices.

$(8 \times 10^9) \div (2 \times 10^6)$

$= 4 \times 10^3$

This is already in standard form because 4 is less than 10.

$(1.2 \times 10^5) \div (2 \times 10^2)$

$= 0.6 \times 10^3$

$= 6 \times 10^2$

This is not in standard form because 0.6 is less than 1.

Adding and subtracting standard form

To add and subtract with standard form we must convert out of standard form into ordinary numbers first and then add/subtract.

$(8.1 \times 10^5) + (2 \times 10^3)$

$= 810000 + 2000$

$= 812000$

$= 8.12 \times 10^5$

Convert into ordinary numbers.

Add/subtract.

Change back into standard form.

$(2.35 \times 10^5) - (4.1 \times 10^3)$

$= 235000 - 4100$

$= 230900$

$= 2.309 \times 10^5$

Surds

Simplifying surds: Break the number inside the root up into a product of 2 factors, where one of those is the largest possible square number. Then you can simplify.

Note: $\sqrt{25}$ is not a surd as it can be written as 5.

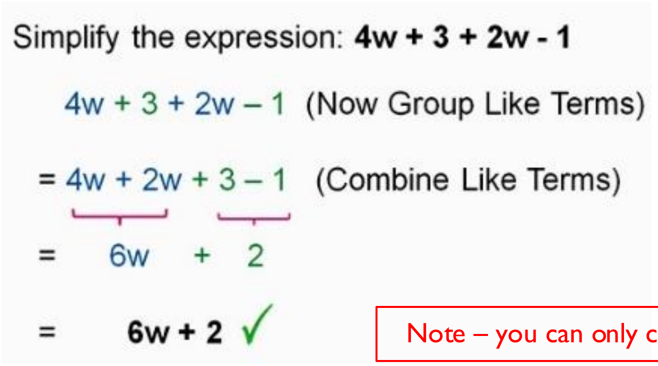
$$\sqrt{50} = \sqrt{25 \times 2}$$
$$= \sqrt{25} \times \sqrt{2}$$
$$= 5 \times \sqrt{2}$$
$$\sqrt{50} = 5\sqrt{2}$$

More on surds in a later chapter…

Knowledge Organiser: 2a Algebra – The Basics

What you need to know:

Collecting like terms



Simplify the expression: **4w + 3 + 2w - 1**

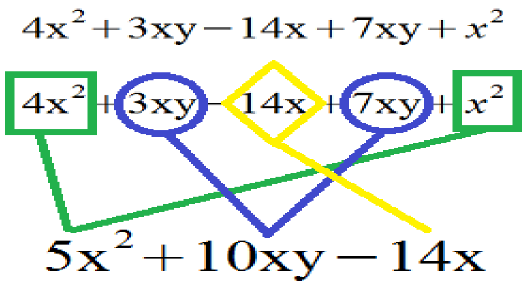
4w + 3 + 2w – 1 (Now Group Like Terms)

= 4w + 2w + 3 – 1 (Combine Like Terms)

$$= \underbrace{4w + 2w}_{6w} + \underbrace{3 - 1}_{2}$$

= 6w + 2

= **6w + 2** ✓



Note – you can only collect terms that have the same power eg $5x + 4x^2 \neq 9x^2$

Substitution

Evaluate (find the value of) the expressions, given that:

$a = 2, \quad b = 3, \quad c = -5$

Note – Always use the correct order of operations

1. $4b = 4 \times 2 = 8$
2. $7b - 3c = (7 \times 3) - (3 \times -5) = 21 - -15 = 21 + 15 = 36$
3. $5b^2 + 1 = 5 \times (3)^2 + 1 = 5 \times 9 + 1 = 45 + 1 = 46$
4. $2c^3 = 2 \times (-5)^3 = 2 \times -125 = -250$
5. $\frac{3ac}{2b} = \frac{3 \times 2 \times -5}{2 \times 3} = \frac{-30}{6} = -5$

For fractions work out the numerator and denominator separately first

Key Terms:

Formula: expresses the relationship between two or more unknown values

Expression: A sentence in algebra that does NOT have an equals sign

Identity: One side is the identically equal to the other side

Substitution: Replace the letter with a given value

Like terms: Variables that are the same are 'like'

Expand: Single brackets – each term inside the bracket is multiplied by the term outside the bracket.
Double brackets – each term in the first bracket is multiplied by all the terms in the second bracket.
Factorise: Putting an expression back into brackets

You need to be able to:

- Identify an expression/equation/formula/identity from a list
- Manipulate and simplify algebraic expressions by collecting 'like' terms
- Substitute numbers into formulae
- Simplify expressions
- Use index notation and the index laws
- Multiply a single term over a bracket and simplify by factorising
- Expand double brackets
- Factorise quadratic expressions

Knowledge Organiser: 2a Algebra – The Basics

What you need to know:

Linear expressions

Expand and simplify where appropriate

1) 7 (3 + a) = 21 + 7a

2) 2(5 + a) + 3(2 + a) = 10 + 2a + 6 + 3a

= 5a + 16

Note – collect like terms to simplify

3) Factorise $9x + 18 = 9(x + 2)$

4) Factorise $6e^2 – 3e = 3e(2e – 1)$

Note – to 'factorise fully' means the HCF will likely be a letter and a number

Quadratic expressions

Expand and simplify:

1) $(p + 2)(2p – 1)$

$= 2p^2 + 4p - p - 2$

$= 2p^2 + 3p - 2$

2) $(p + 2)^2$

$(p + 2)\ (p + 2)$

$= p^2 + 2p + 2p + 4$

$= p^2 + 4p + 4$

Factorise:

3) $x^2 - 2x - 3$

$= (x - 3)(x + 1)$

Factorise and solve:

4) $x^2 + 4x - 5 = 0$

$(x - 1)(x + 5) = 0$

Therefore the solutions are:

Either $(x - 1) = 0$

$x = 1$

Or $(x + 5) = 0$

$x = -5$

Index Laws

Simplify the following: $\boxed{a^3 \times a^4}$

If we start by writing it out in full:

$a^3 = a \times a \times a$

$a^4 = a \times a \times a \times a$

$\therefore a \times a \times a \times a \times a \times a \times a = a^7$

To multiply together two identical values or variables (letters) that are presented in index form, add the powers.

$d^8 \times d^2 = d^{10}$ $e^{-3} \times e^5 = e^2$ $f \times f^3 = f^4$

Simplify the following: $\boxed{\dfrac{m^5}{m^3}}$

If we start by writing it out in full:

$m^5 = m \times m \times m \times m \times m$

$m^3 = m \times m \times m \times m \times m$

$$\therefore \frac{\cancel{m} \times \cancel{m} \times \cancel{m} \times m \times m}{\cancel{m} \times \cancel{m} \times m} = m^2$$

$p^6 \div p^3 = p^3$ $\dfrac{s^3}{s^7} = s^{-4}$

To divide two identical values or variables (letters) that are presented in index form, subtract the powers

Simplify the following:

$(m^7)^3 = m^{21}$ $(n^6)^{-4} = n^{-24}$

To raise a value or variable (letter) presented in index form to another index, multiply the powers together

Reminders:

$a^1 = a$

$a^0 = 1$

Anything to the power of zero = 1

Difference of two squares

$a^2 - b^2 = (a + b)(a - b)$

Index Laws - Negative

$\boxed{a^{\frac{1}{2}} = \sqrt{a}}$ $\boxed{a^{-m} = \dfrac{1}{a^m}}$

$9^{\frac{1}{2}} = \sqrt{9} = 3$ or -3

$64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$

$7^{-1} = \dfrac{1}{7}$

$8^{-4} = \dfrac{1}{8^4} = \dfrac{1}{4096}$

Knowledge Organiser: 2a Algebra – The Basics

What you need to know:

Solving Equations

Unknown on one side

Solve 2x + 1 = 9

-1 2x = 8 -1

÷2 x = 4 ÷2

Solve 3(y – 7) = 9

Always expand the bracket first

3y – 21 = 9

+21 3y = 30 +21

÷3 y = 10 ÷3

You can check your answers by substituting them back into the question

Unknowns on both side

Solve 2d - 7 = 5d - 10

Start by subtracting the smallest amount of the variable from both sides

-2d -7 = 3d - 10 -2d

+10 3 = 3d +10

÷3 d = 1 ÷3

Solve 3(2t + 4) = 2(2 - t)

6t + 12 = 4 – 2t

+2t 8t + 12 = 4 +2t

-12 8t = -8 -12

÷8 t = -1 ÷8

Set up equations from word problems

Jenny, Kenny, and Penny together have 51 marbles. Kenny has double as many marbles as Jenny has, and Penny has 12. How many does Jenny have?

Set up an equation then solve

Jenny's + Kenny's + Penny's = 51

n + 2n + 12 = 51

3n + 12 = 51

-12 3n = 39 -12

÷3 n = 13 ÷3

You need to be able to:

- Solve linear equations
- Derive a formula and set up equations from worded problems
- Change the subject of a formula
- Use iteration to find approximate solutions to equations

Key Terms:

Solve: Find a numerical value that satisfies the equation

Inverse operation: The operation that reverses the effect of another operation e.g. subtraction in the inverse of addition

Rearrange: Change the variable before the "=" using inverse operations

Knowledge Organiser: 2a Algebra – The Basics

What you need to know:

Rearranging Formulae

Change the order of the terms so 'u' is on its own

Make u the subject: $v = u + at$

-at

-at

$v - at = u$

so $\underline{u = v - at}$

Make m the subject: I = $mv - mu$

If the letter appears twice you will need to factorise

÷ (v – u)

$I = m(v - u)$

÷ (v – u)

$I \div (v - u) = m$

$$\boxed{m = \frac{I}{v - u}}$$

Iteration

Starting with $x_0 = 0$ use the iteration formula

$$\boxed{x_{n+1} = \frac{2}{x_n^2 + 3}}$$

3 times to find an estimate to the solution.

Calculate the values of x_1, x_2, x_3 to find an estimate for the solution to $x^3 + 3x = 2$

$$x_{0+1} = \frac{2}{0^2 + 3} = 0.\dot{6}$$

We substitute this value into the next step.

$$x_{1+1} = \frac{2}{0.\dot{6}^2 + 3} = 0.5806451613$$

$$x_{2+1} = \frac{2}{(0.58\ldots)^2 + 3} = 0.5993140006$$

An estimate of the solution is 0.6 because all of the solutions round to 1d.p.
x_1, x_2, x_3 are increasingly more accurate values for **one** solution of the equation

Key Concepts

Iteration is the **repetition** of a mathematical process applied to the result of a previous application, typically as a means of **obtaining successively closer approximations** to the solution of a problem.

Equations of motion

$v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

s – displacement
u – initial velocity
v – final velocity
a – acceleration
t - time

You will be asked to use these equations and substitute into them. You do not need to memorise them.

Knowledge Organiser: 2b Sequences

Key Concepts

Arithmetic sequences increase or decrease by a common amount each time.

Quadratic sequences have a common 2nd difference.

Fibonacci sequences Add the two previous terms to get the next term

Geometric series has a common multiple between each term

Examples

Linear sequences:

4 , 7, 10, 13, 16…..

a) State the nth term

$3n + 1$

Difference

The 0th term

b) What is the 100th term in the sequence?

$3n + 1$

$3 \times 100 + 1 = 301$

c) Is 100 in this sequence?

$3n + 1 = 100$

$3n = 99$

$n = 33$

Yes as 33 is an integer.

Quadratic sequences:

$a + b + c$	3	9	19	33	51	
$3a + b$	6	10	14	18		First difference
$2a$	4	4	4			Second difference

$2a = 4$
$a = 2$

$3a + b = 6$
$3 \times 2 + b = 6$
$b = 0$

$a + b + c = 3$
$2 + 0 + c = 3$
$c = 1$

$2n^2 + 0n + 1 \rightarrow 2n^2 + 1$

Key Words

Linear
Quadratic
Arithmetic
Geometric
Sequence
Nth term

A) 1, 8, 15, 22, ….

1) Find the nth term   b) Calculate the 50th term   c) Is 120 in the sequence?

B) Find the nth term for:

1) 5, 12, 23, 38, 57, …   2) 3, 11, 25, 45, 71, ….

ANSWERS: A1) 7n – 6 2) 344 3) 18 so yes as n is an integer B1) $2n^2 + n + 2$ 2) $3n^2 – n + 1$

Knowledge Organiser: 3a Averages and the Range

What you need to know:

Two-Way Tables

This **two-way table** gives information on how 100 students travelled to school. Complete the two-way table

	Walk	Car	Other	Total
Boy	15	25	14	54
Girl	22	8	16	46
Total	37	33	30	100

Total Boys

Total Girls

Total number of people

Stem and Leaf Diagrams

A stem and leaf diagram can be used for large data sets. You can then interpret the data including finding averages and range.

Key 7|6 = 67cm

Female		Male
8 5	7	6 7 9
7 5 4 3 0	8	3 5 7 8
9 8 6 1	9	2 3 5 7 8
	10	1 3 7

Order the leaves from smallest to biggest

Key 7|6 = 76cm

Always include a key

Smallest girl = 75cm Smallest Boy = 76cm

Tallest girl = 99cm Tallest Boy = 107cm

Reverse Mean

A hockey team scored the following number of goals in 6 games:

2	3	4	1	0	1

The mean of the goals scored in seven games was 2. How many goals were scored in the seventh game?

Mean = add all together divide by how values

$$\frac{2+3+4+1+0+1+x}{7} = 2 \longrightarrow \frac{11+x}{7} = 2 \longrightarrow \underline{x = 3}$$

Solve the equation to find the missing value

Advantages and Disadvantages

Average	Advantage	Disadvantage
Mode	Can be used for qualitative data Easy to obtain	There can be more than one mode or even no mode
Median	Not affected by very large or small values	Can be time consuming when there is a lot of data
Mean	Takes into account all of the data	Very small or very large values affect the mean

Key Terms:

Frequency - the number of pieces of data we have.

Grouped Data - If we have a large spread of data, we put it into categories (classes) to make the data easier to display or analyse

You need to be able to:

- Recognise types of data
- Design and use two-way tables
- Calculate the mean, mode, median and range
- Construct and interpret stem and leaf diagrams
- Know the advantages and disadvantages of averages
- Interpret and find averages from frequency tables
- Find the range, modal class, interval containing the median and estimate the mean from grouped data

Reminder:

Mean - Add up the values you are given and divide by the number of values you have.

Median - The median is the middle value, when your data is in order.

Mode - It is the value or item with the greatest frequency.

Range - This is the difference between the largest and smallest values.

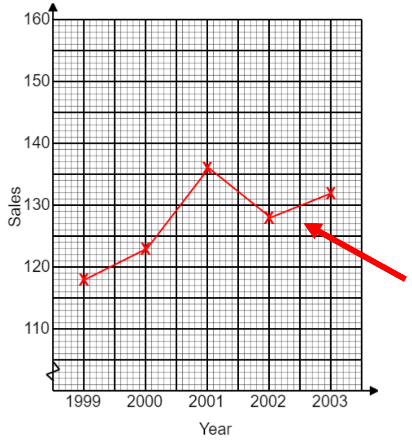
Knowledge Organiser: 3b Representing and Interpreting Data and Scatter Graphs

What you need to know:

Time-series Graphs

Plot the following sales information on the graph below and describe the overall trend:

Year	1999	2000	2001	2002	2003
Sales	118	123	136	128	132



Step 1 – Label the x and y axes, and use an appropriate scale

Try to fill the graph paper

Step 2 – Plot each point onto the graph

Double check what one square represents

Step 3 – Join up each point with a straight line

Step 4 – Identify the overall pattern shown = generally increasing (rising trend)

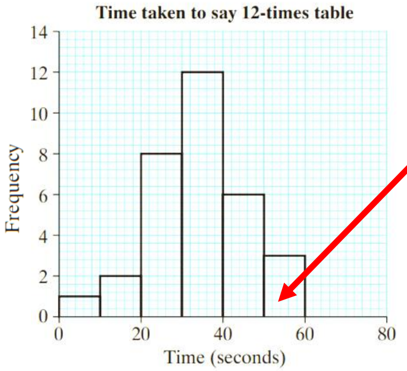
Visualising a line of best fit through the plotted points can help you to see the overall trend

Histograms with **Equal** Class Intervals

A group of 32 students were asked to say the 12-times table as fast as possible.

a) Draw a histogram for the following data:

Time, t (s)	Frequency
$0 < t \leqslant 10$	1
$10 < t \leqslant 20$	2
$20 < t \leqslant 30$	8
$30 < t \leqslant 40$	12
$40 < t \leqslant 50$	6
$50 < t \leqslant 60$	3



See *Cumulative Frequency, Box Plots, and Histograms* for more on drawing histograms

No gaps between bars

$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Class Width}}$$

Key Terms:

Discrete data: countable data that can be categorised *e.g. Shoe size, eye colour*

Continuous data: data that is measured and can take any value e.*g. Height, time, temperature*

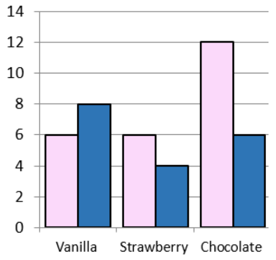
Qualitative data: text-based data that describes something *e.g. colours, race*

Quantitative data: numerical data *e.g. age, height, temperature*

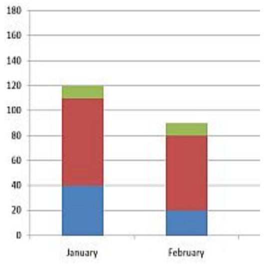
Frequency: the number of occurrences of an event

Extrapolate: to predict values from outside the range of data

Bar Charts



Comparative bar charts show data side by side



Compound bar charts show data stacked

You need to be able to:

- Know what chart to use for different types of data sets
- Draw and interpret all types of bar charts, pie charts, frequency polygons, line graphs, and time-series graphs
- Recognise simple patterns in graphs and charts (e.g. seasonal patterns)
- Estimate the median from a histogram with equal class intervals
- Compare averages of two distributions
- Predict future values from a time-series graph

Knowledge Organiser: 3b Representing and Interpreting Data and Scatter Graphs

What you need to know:

Pie Charts

Use the data in the following table to draw a pie chart

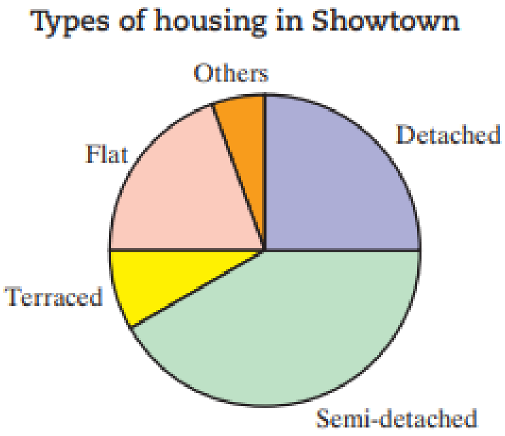
House Type	Frequency	Angle
Detached	18	18 x 5° = 90°
Semi-detached	30	30 x 5° = 150°
Terraced	6	6 x 5° = 30°
Flat	14	14 x 5° = 70°
Other	4	4 x 5° = 20°

Total = 72

Finding angles:
Step 1 – Divide 360° by your total frequency to find how many ° represents one house
= 360 ÷ 72 = 5°

Step 2 – Multiply the frequency for each house type by the ° per house

Drawing the pie chart:
Step 1 – Draw a circle using a compass, and draw a vertical line from the centre to the top
Step 2 – Using a protractor, measure and draw each angle
Step 3 – Label each section of the pie chart
Step 4 – Give your pie chart a suitable title

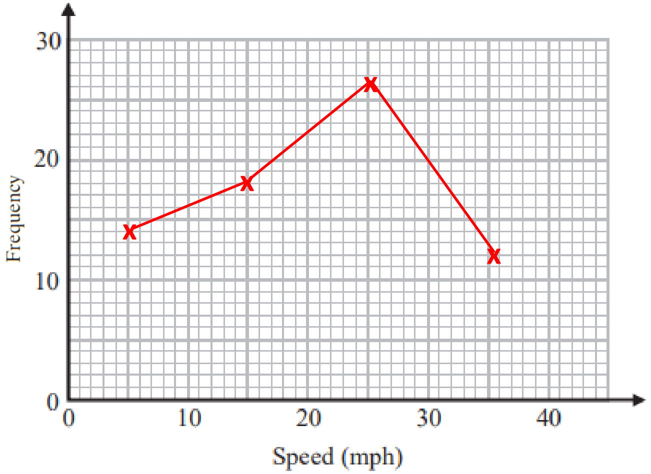


Drawing Frequency Polygons

This table gives information about the speeds of 70 cars.

Speed (*s* mph)	Frequency (f)	Midpoint
$0 < L \le 10$	14	5
$10 < L \le 20$	18	15
$20 < L \le 30$	26	25
$30 < L \le 40$	12	35

a) Draw a frequency polygon for this information.



Step 1 – Find the **midpoint** of each class interval
Step 2 – Label your axes and choose an appropriate scale
Step 3 – Plot each point at the midpoint for that interval
Step 4 – Connect each point with a straight line **using a ruler**

Do not extend the line beyond the points you have plotted

b) Identify the interval with the median speed

$$Median\ position = \frac{Total\ frequency+1}{2}$$

Step 1 – Identify the median car

Median car = 71 ÷ 2
= 35.5

Step 2 – Which bracket does this car fall into?

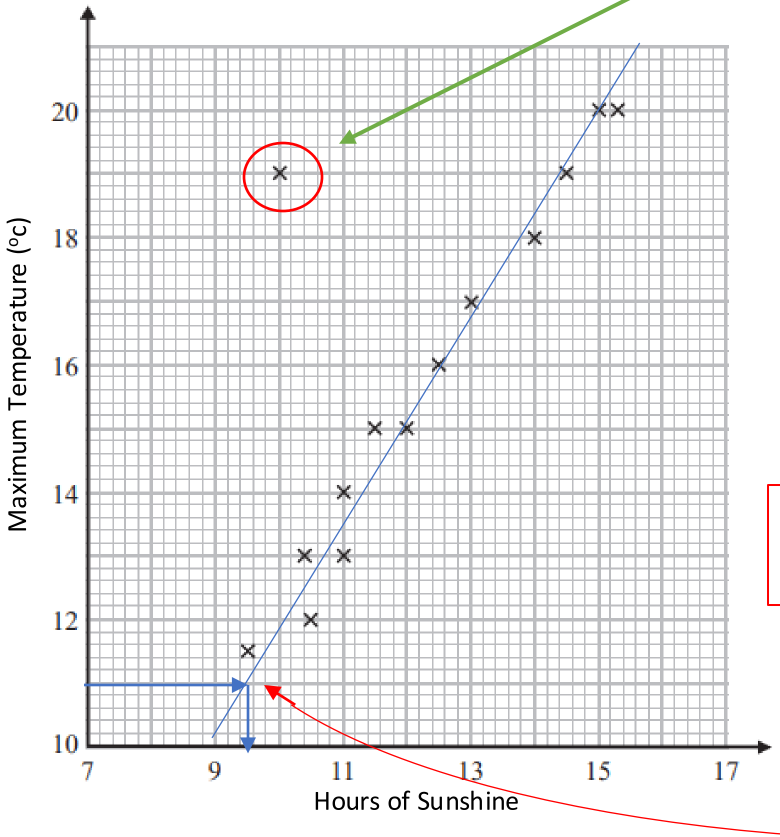
35.5 occurs in the 20<*L*≤30 group, so the **median interval is 20 < L ≤ 30**

Knowledge Organiser: 3b Representing and Interpreting Data and Scatter Graphs

What you need to know:

Scatter Graphs

This scatter graph shows the maximum temperature and the number of hours of sunshine in 14 British towns in one day.



Scatter Graphs - Outliers and Correlation

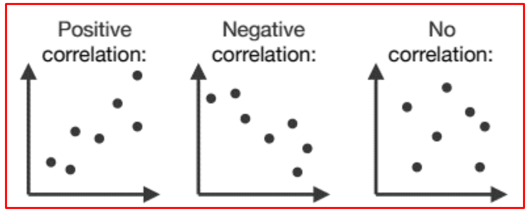
Identify the coordinates of the outlier.

= (10, 19)

An outlier is a value that doesn't fit the pattern of the data

What type of correlation does the remaining data show?

= Positive correlation



Scatter Graphs – Correlation and Causation

A student looks at the graph and says, "This graph shows that sunshine causes higher temperatures". Is this true? Give a reason.

Correlation does not imply causation. While it may look like variables are related, there may be something else responsible for the data points.

= No, although the graph shows a positive correlation, this does not mean there is a causal link between hours of sunshine and maximum temperature

Scatter Graphs – Explaining Patterns

A weatherman says, "Temperatures are higher in towns that have more sunshine". Is this supported by the scatter graph?

= Yes, the majority of points for high temperature appear when there are more hours of sunshine.

Interpolation and Extrapolation

Interpolation – making a prediction of a value that falls within the range of your data. This is more accurate.

Extrapolation – making a prediction of a value that falls outside the range of your data. This is less accurate.

Another town had a maximum temperature of 11°c that day. Use a line of best fit to estimate the hours of sunshine at this town.

Step 1 – Draw a line of best fit
Step 2 – Draw a line along from 11°c and down from the line of best fit

= 9.5 hours

Comment on the reliability of your prediction.

= This is not a reliable estimate because the data has been extrapolated to cover this temperature

Knowledge Organiser: 4a
Fractions

Key Concepts

Equivalent fractions have the same value as one another.

Eg. $\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$

A number multiplied by its **reciprocal** gives the answer of 1. Or the reciprocal of a number is 1 over the number.

Eg. $\frac{1}{8}$ is the reciprocal of 8.

$\frac{2}{5}$ is the reciprocal of $\frac{5}{2}$

Examples

$$1\frac{2}{3} + 2\frac{1}{4} \qquad\qquad 2\frac{2}{3} - 1\frac{1}{4}$$

Convert into an improper fraction

$$= \frac{5}{3} + \frac{9}{4} \qquad\qquad = \frac{8}{3} - \frac{5}{4}$$

Find a common denominator

$$= \frac{20}{12} + \frac{27}{12} \qquad\qquad = \frac{32}{12} - \frac{15}{12}$$

$$= \frac{47}{12} \qquad\qquad = \frac{17}{12}$$

Convert back into a mixed number

$$= 3\frac{11}{12} \qquad\qquad = 1\frac{5}{12}$$

$$1\frac{1}{3} \times 2\frac{3}{4}$$

×

$$= \frac{4}{3} \times \frac{11}{4}$$

×

$$= \frac{44}{12}$$

$$= 3\frac{8}{12}$$

$$2\frac{1}{3} \div 1\frac{3}{5}$$

$$= \frac{7}{3} \div \frac{8}{5}$$

Find the reciprocal of the second fraction….

$$= \frac{7}{3} \times \frac{5}{8}$$

…and multiply

$$= \frac{35}{24}$$

$$= 1\frac{11}{24}$$

Key Words

Fraction
Equivalent
Reciprocal
Numerator
Denominator
Improper/Top heavy
Mixed number

Calculate:

1) $1\frac{2}{3} + 2\frac{3}{4}$

2) $3\frac{3}{4} - 1\frac{1}{3}$

3) $3\frac{1}{5} \times 1\frac{2}{3}$

4) $1\frac{3}{5} \div 2\frac{7}{10}$

What is the reciprocal of:

5) $\frac{2}{3}$

6) 9

7) 0.75

ANSWERS A 1) $4\frac{5}{12}$ 2) $2\frac{5}{12}$ 3) $5\frac{1}{3}$ 4) $\frac{16}{27}$ 5) $\frac{3}{2}$ 6) $\frac{1}{9}$ 7) $\frac{4}{3}$

Knowledge Organiser: 4a
Percentage Change and Reverse Percentages

Key Concepts

Calculating percentages of an amount without a calculator:

10% = divide the value by 10
1% = divide the value by 100

Calculating percentages of an amount with a calculator:

Amount × percentage
as a decimal

Calculating percentage increase/decrease:

Amount × (1 ± percentage
as a decimal)

Examples

Percentage change:

A dress is reduced in price by 35% from £80. What is it's **new price**?

$Value \times (1 - percentage\ as\ a\ decimal)$
= $80 \times (1 - 0.35)$
= £52

A house price appreciates by 8% in a year. It originally costs £120,000, what is the **new value** of the house?

$Value \times (1 + percentage\ as\ a\ decimal)$
= $120{,}000 \times (1 + 0.08)$
= £129,600

Reverse percentages: This is when we are trying to find out the original amount.

A pair of trainers cost £35 in a sale. If there was 20% off, what was the **original price** of the trainers?

$Value \div (1 - 0.20)$
= $35 \div 0.8$
= £43.75

A vintage car has increased in value by 5%, it is now worth £55,000. What was it worth **originally**?

$Value \div (1 + 0.05)$
= $55{,}000 \div 1.05$
= £52,380.95

Key Words

Percent
Increase/decrease
Reverse
Multiplier
Inverse

1a) Decrease £500 by 6%
b) Increase 70 by 8.5%
2) A camera costs £180 in a 10% **sale**. What was the **pre-sale** price
3) The cost of a holiday, including **VAT** at 20% is £540. What is the **pre-VAT** price?

ANSWERS A 1a) £470 b) £75.95 2) £200 3) £450

Knowledge Organiser: 4a
Compound Interest and Depreciation

Key Concepts

We use **multipliers** to increase and decrease an amount by a particular percentage.

Percentage increase:

$$Value\ \times (1 + percentage\ as\ a\ decimal)$$

Percentage decrease:

$$Value\ \times (1 - percentage\ as\ a\ decimal)$$

Appreciation means that the value of something is going up or increasing.

Depreciation means that the value of something is going down or reducing.

Per annum is often used in monetary questions meaning **per year.**

Key Words

Percent
Appreciate
Depreciate
Interest
Annum
Compound
Multiplier

Examples

Compound interest:

Joe invest £400 into a bank account that pays 3% **compound interest** per annum. Calculate how much money will be in the bank account after 4 years.

$Value$
$\times (1 + percentage\ as\ a\ decimal)^{years}$
= $400\ \times (1 + 0.03)^4$
= $400\ \times (1.03)^4$
= £450.20

Compound depreciation:

The original value of a car is £5000. The value of the car **depreciates** at a rate of 7.5% per annum. Calculate the value of the car after 3 years.

$Value\ \times (1 - percentage\ as\ a\ decimal)^{years}$
= $5000\ \times (1 - 0.075)^3$
= $5000\ \times (0.925)^3$
= £3957.27

1) Jane invests £670 into a bank account that pays out 4% compound interest per annum. How much will be in the bank account after 2 years?
2) A house has decreased in value by 3% for the past 4 years. If originally it was worth £180,000, how much is it worth now?

ANSWERS A 1) £724.67 2) £159352.71

Knowledge Organiser: 4b
Dividing an Amount into Ratios

Key Concepts

An amount can be divided into a given ratio.

Red : Green
1 : 3

For every 1 red there are 3 greens.

A ratio can be converted into fractions.

Red : Green
1 : 3

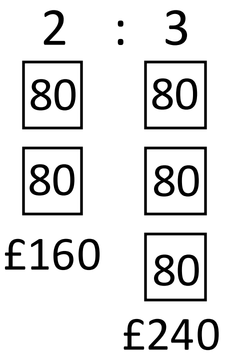
$\frac{1}{4}$ are red and $\frac{3}{4}$ are green.

Examples

A woman has £400. She is going to split her money between her two children in the ratio 2:3. How much does each child receive?

No. of boxes (2+3)

$400 \div 5$
$= 80$



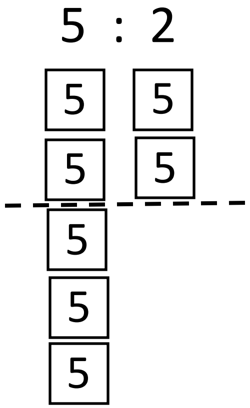
Child 1 receives £160 and Child 2 receives £240.

There are boys and girls at a party in the ratio 5:2.
There are 15 more boys than girls.
Calculate the number of people at the party.

No. of extra Boxes (5-2)

$15 \div 3$
$= 5$

7×5
$= 35$ people



Key Words

Ratio
Divide
Parts

1) Ann made some cakes. She made vanilla cakes and chocolate cakes in the ratio 2:9. What fraction of the cakes were chocolate?
2) Share £25 in the ratio 7:3
3) Katy and Becky share some money in the ratio 2:1. Katy receives £10 more than Becky. How much do they each receive?
4) Claire and John share some money in the ratio 3:2. Claire receives £18. How much does John receive?

ANSWERS 1) $\frac{9}{11}$ 2) £17.50, £7.50 3) £20, £10 4) £12

Knowledge Organiser: 4b
Ratio and Direct Proportion

Key Concepts

To calculate the **value** for a single item we can use the **unitary method**.

When working with best value in monetary terms we use:

$$Price\ per\ unit = \frac{price}{quantity}$$

In recipe terms we use:

$$Weight\ per\ unit = \frac{weight}{quantity}$$

Examples

If 20 apples weigh 600g. How much would 28 apples weigh?

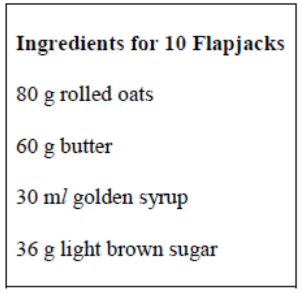
600 ÷ 20 = 30g weight of 1 apple
28 × 30 = **840g**

Box A has 8 fish fingers costing £1.40.
Box B has 20 fish fingers costing £ 3.40.
Which box is the better value?

$$A = \frac{\text{£}1.40}{8} = \text{£}0.175 \qquad B = \frac{\text{£}3.40}{20} = \text{£}0.17$$

Therefore Box B is better value as each fish finger costs less.

The recipe shows the ingredients needed to make 10 Flapjacks.
How much of each will be needed to make 25 flapjacks?



Ingredients for 10 Flapjacks

80 g rolled oats

60 g butter

30 ml golden syrup

36 g light brown sugar

Method 1: Unitary

80 ÷ 10 = 8
8 × 25 = **200g**

30 ÷ 10 = 3
3 × 25 = **75g**

60 ÷ 10 = 6
6 × 25 = **150g**

36 ÷ 10 = 3.6
3.6 × 25 = **90g**

Method 2: 5 flapjacks

80 ÷ 2 = 40
40 × 5 = **200g**

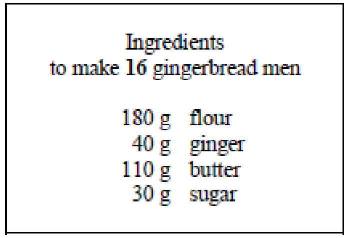
30 ÷ 2 = 15
15 × 5 = **75g**

60 ÷ 2 = 30
30 × 5 = **150g**

36 ÷ 2 = 18
18 × 5 = **90g**

Key Words

Unitary
Best Value
Proportion
Quantity



Ingredients
to make 16 gingerbread men

180 g flour
40 g ginger
110 g butter
30 g sugar

1) How much will we need to make 24 gingerbread men?

2) Packet A has 10 toilet rolls costing £3.50.
Packet B has 12 toilet rolls costing £3.60.
Which is better value for money?

3) If 15 oranges weigh 300g. What will 25 oranges weigh?

ANSWERS 1) 270g flour, 60g ginger, 165g butter, 45g sugar 2) Packet B 30p per roll 3) 500g

Knowledge Organiser: 4b
Direct and Inverse Proportion

Key Concepts

Variables are **directly proportional** when
the **ratio is constant** between the quantities.

Variables are **inversely proportional** when **one quantity increases in proportion to the other decreasing.**

Examples

Direct proportion:

Value of A	32	P	56	20	72
Value of B	20	30	35	R	45

Ratio constant: $20 \div 32 = \frac{5}{8}$

From A to B we will multiply by $\frac{5}{8}$.

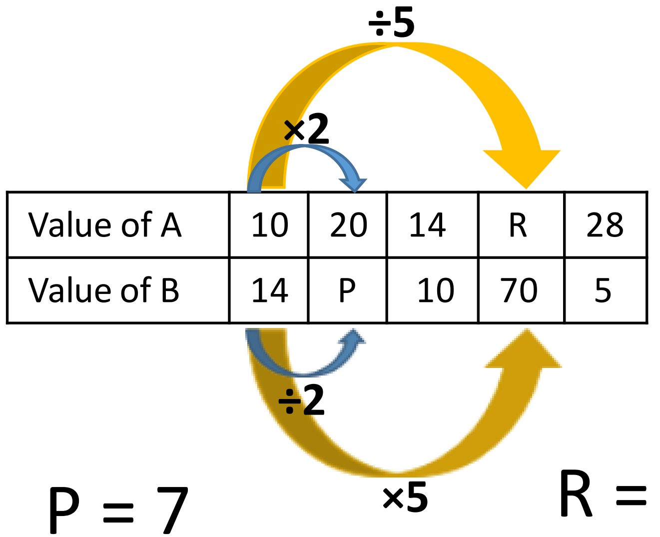
From B to A we will divide by $\frac{5}{8}$.

$P = 30 \div \frac{5}{8} = 48$

R= 20

$\times \frac{5}{8} = 12.5$

Inverse proportion:



Value of A	10	20	14	R	28
Value of B	14	P	10	70	5

P = 7

R = 2

Key Words

Direct
Inverse
Proportion
Divide
Multiply
Constant

Complete each table:

1) Direct proportion

Value of A	5	P	22
Value of B	9	28.8	Q

2) Inverse proportion

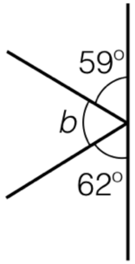
Value of A	4	P	18
Value of B	9	3	Q

ANSWERS 1) P = 16, Q = 39.6 2) P = 12, Q = 2

Knowledge Organiser: 5a Angles, Polygons and Parallel Lines

What you need to know:

Straight lines and around a point

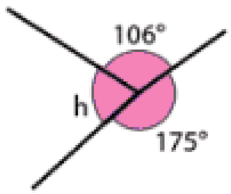


To calculate b:

62° + 59° = 121°

180° - 121° = 59°

Add the angles we know together first then subtract from 180°.



To calculate h:

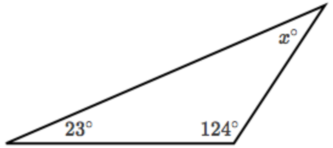
175° + 106° = 281°

360° - 281° = 79°

Add the angles we know together first then subtract from 360°.

Triangles

Angles in a triangle add up to 180°.

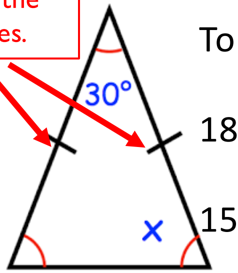


To calculate x:

23° + 124° = 147°

180° - 147° = 33°

This means that these 2 sides are equal and the 2 angles at the end of the sides.



Subtract from 180°.

To calculate x:

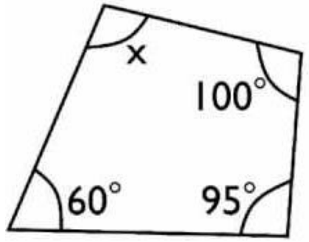
180° - 30° = 150°

150° ÷ 2 = 75°

Divide by 2 to calculate 1 angle.

Quadrilaterals

Angles in a quadrilateral add up to 360°.



Add the angles we know together first then subtract from 360°.

To calculate x:

60° + 100° + 95° = 255°

360° - 255° = 105°

Key Terms:

Quadrilateral: A 2D shape with four sides.

Polygon: A 2D shape.

Regular Polygon: A shape where all of the sides are equal length.

Irregular Polygon: A shape where all of the sides are not equal lengths.

Isosceles: A triangle that has 2 equal sides and 2 equal angles.

Equilateral: A triangle where all of the sides and angles are equal.

Vertically opposite: The 2 angles that are facing each other are equal where 2 lines cross.

Parallel: Always the same distance apart and never touching.

Perpendicular: Meet at right angles (90^0).

Interior angle: An angle inside a shape.

Exterior angle: The angle between any side of a shape, and a line extended from the next side.

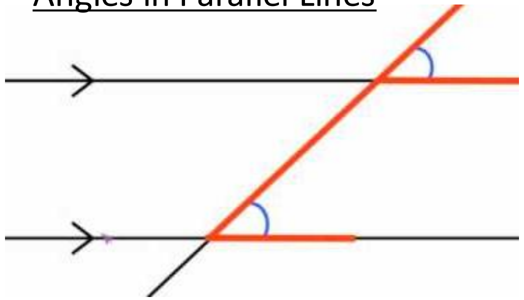
You need to be able to:

- Classify quadrilaterals and triangles by their geometric properties.
- Calculate missing angles in triangles, quadrilaterals and using the rules of vertically opposite angles.
- Calculate missing angles inside parallel lines and explain using the correct terminology.
- Combine basic angle facts with parallel line facts to solve problems.
- Calculate interior and exterior angles of a regular or irregular polygon and calculate the number of sides.

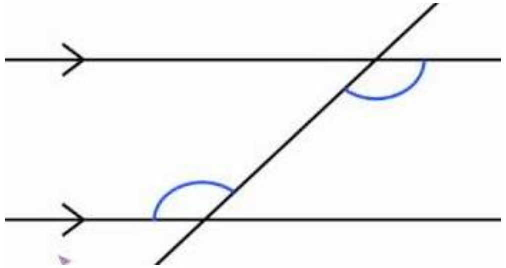
Knowledge Organiser: 5a Angles, Polygons and Parallel Lines

What you need to know:

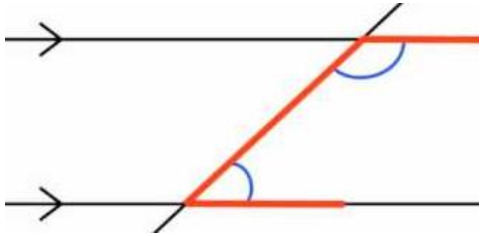
Angles in Parallel Lines



Corresponding angles are equal.



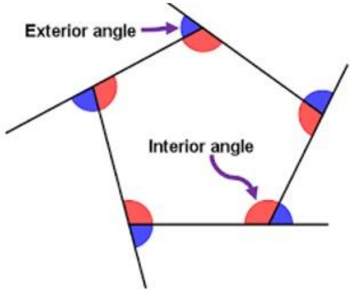
Alternate angles are equal.

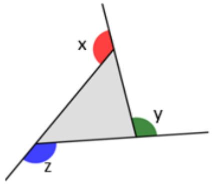


Co-interior angles sum to 180°.

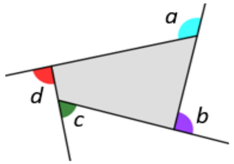
Exterior Angles in Polygons

Exterior angles in a polygon sum to 360°.





$x + y + z = 360°$



$a + b + c + d = 360°$

The exterior angle of a regular polygon is calculated using: 360 ÷ n

n = number of sides

Interior Angles in Regular Polygons

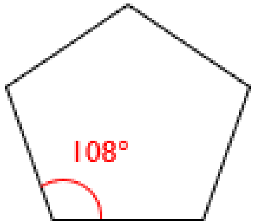
Calculate the size of one interior angle in a pentagon.

Step 1 – Calculate the sum of the interior angles

Sum of interior angles = (n – 2) x 180

n = number of sides

Step 2 – Divide by the number of sides



(5 – 2) x 180 = 540

540 ÷ 5 = 108°

Calculating the number of sides

Example – A regular polygon has exterior angles of 20°. How many sides does it have?

Exterior angle = 360 ÷ number of sides

Number of sides = 360 ÷ 20 = 18

18 sides

Rearrange: number of sides = 360 ÷ angle

Interior and exterior angles key formulae:

Sum of the Interior Angles	$180(n – 2)$
Sum of the Exterior Angles	Always 360°!
Each Interior Angle	$\frac{180(n – 2)}{n}$
Each Exterior Angle	$\frac{360°}{n}$

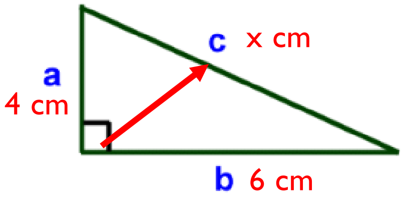
Knowledge Organiser: 5b Pythagoras’ Theorem and Trigonometry

What you need to know:

Pythagoras’ Theorem - Hypotenuse

You should always label the hypotenuse first.
This is the side facing the right angle.

$$a^2 + b^2 = c^2$$



1) Substitute your values into the formulae:

$$4^2 + 6^2 = x^2$$

2) Work out the values that you can.

$$16 + 36 = x^2$$

$$52 = x^2$$

3) Now use inverse operations to isolate x.

$$52 = x^2$$

$$(\surd) \quad (\surd)$$

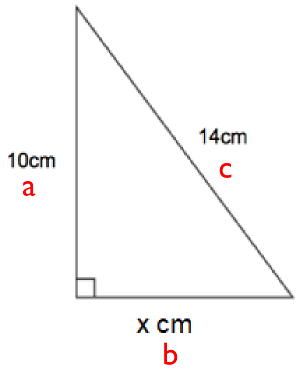
$$\sqrt{52} = x$$

7.211102551 cm = x or 7.21 to 3 s.f

This is surd form.
Sometimes you will be asked to leave your answer like this.

Pythagoras’ Theorem – Shorter Sides

$$a^2 + b^2 = c^2$$



Sometimes you are asked to calculate the shorter sides, see below.

1) Substitute your values into the formulae:

$$10^2 + x^2 = 14^2$$

2) Work out the values that you can.

$$100 + x^2 = 196$$

3) Now use inverse operations to isolate x.

$$100 + x^2 = 196$$

$$(-100) \qquad (-100)$$

$$x^2 = 96$$

$$(\surd) \quad (\surd)$$

$$\sqrt{96} = x$$

x = 9.797958971 cm or 9.80cm to 3 s.f

You need to get the numbers on one side, the x on its own.
An extra step is needed.

Key Terms:

Hypotenuse: The longest side in a right-angled triangle.

Opposite: The side facing the angle in a right-angled triangle.

Adjacent: The side next to the angle given in a right-angled triangle.

Square number: The result when you multiply a number by itself.

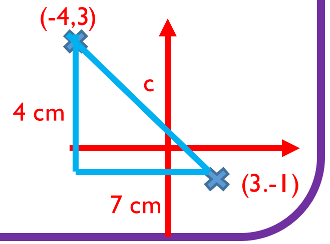
Inverse operation: The operation that reverses the effect of another operation.

Sine, Cosine, Tangent: Trigonometric ratios using the sides of a triangle, relating to buttons on the calculator.

The distance between two points

Find the distance between (3,-1) and (-4,3)
1)Sketch coordinates on an axis.
2)Join as a right-angled triangle.
3)Find the lengths of the straight sides.
4)Use Pythagoras to find the hypotenuse.

42 + 72 = c^2
65 = c^2
$\sqrt{65}$ = c
c = 8.062257748



You need to be able to:

- Identify the hypotenuse in a right-angled triangle.
- Use Pythagoras’ theorem to find the hypotenuse and shorter sides.
- Use Pythagoras’ theorem to solve 3D problems.
- Use trigonometry to find lengths and angles in right angled triangles.
- Use right angled trigonometry to solve 3D problems.
- Recall exact trigonometric values.

Knowledge Organiser: 5b Pythagoras' Theorem and Trigonometry

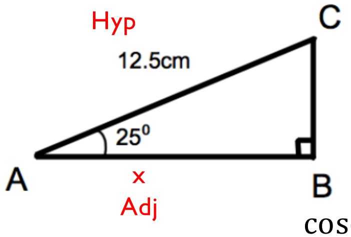
What you need to know:

SOH **CAH** **TOA**

SINE = $\frac{\text{OPP}}{\text{HYP}}$ COSINE = $\frac{\text{ADJ}}{\text{HYP}}$ TANGENT = $\frac{\text{OPP}}{\text{ADJ}}$

Trigonometry – Finding a side 1

Calculate the length of AB.



Step 1 – Label the sides you need as O, A or H.
Step 2 – Use this to decide which trig ratio to use.
Step 3 – Substitute the given values into the formula.
Step 4 – Use inverse operations to rearrange & isolate x.

$$\cos(25) = \frac{x}{12.5}$$

The inverse of dividing by 12.5 is multiplying.

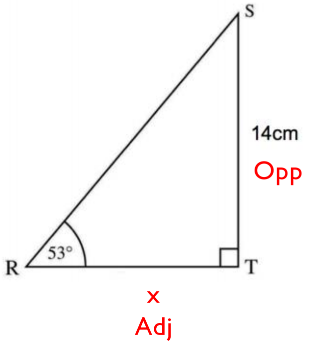
$$\cos(25) \times 12.5 = x$$

$$x = 11.32884734\ cm$$

Don't round your answer, you get no marks for this!

Trigonometry – Finding a side 2

Calculate the length of RT.



Step 1 – Label the sides you need as O, A or H.
Step 2 – Use this to decide which trig ratio to use.
Step 3 – Substitute the given values into the formula.
Step 4 – Use inverse operations to rearrange & isolate x.

$$\tan(53) = \frac{14}{x}$$

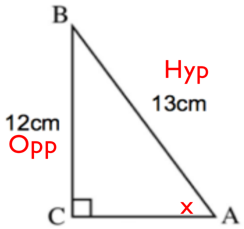
$$x = \frac{14}{\tan 53}$$

$$x = 10.5497567\ cm$$

When the unknown value is on the bottom of the fraction the x and tan(53) swap places.
This is because you have multiplied the LHS by x and the divided the RHS by tan(53).

Trigonometry – Finding an angle

Calculate the size of angle BAC.



Step 1 – Label the sides you need as O, A or H.
Step 2 – Use this to decide which trig ratio to use.
Step 3 – Substitute the given values into the formula.
Step 4 – Use inverses to rearrange & isolate x.

$$\sin(x) = \frac{12}{13}$$

$$x = \sin^{-1}\left(\frac{12}{13}\right)$$

$$x = 67.38013505°$$

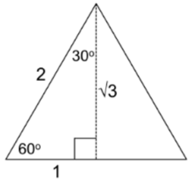
The inverse of sin, cos and tan are $\sin^{-1}$, $\tan^{-1}$, $\cos^{-1}$. They are found by pressing shift sin on your calculator.

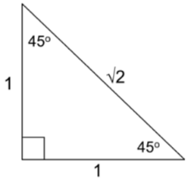
I have not labelled the third side as it has no information on, and I am not trying to calculate it.

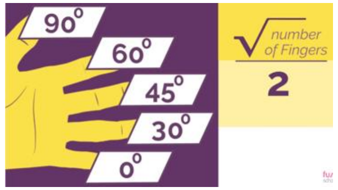
Trigonometry – Exact values

For your exam you will need to learn the following values.
(Use the hand trick or triangles to help you learn them)

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	–



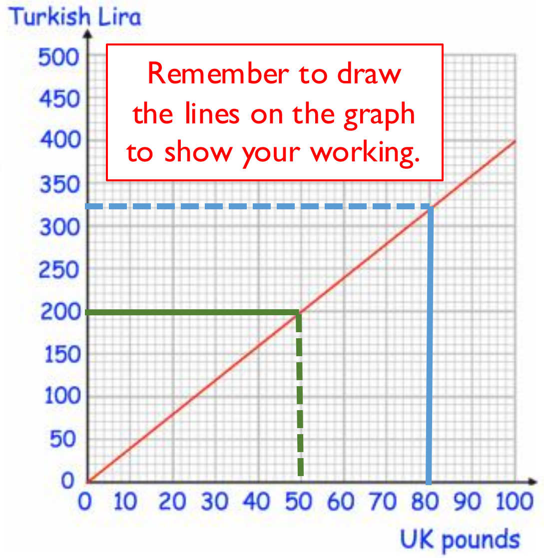




Knowledge Organiser: 6a Graphs: The Basics and Real-Life Graphs

What you need to know:

Conversion graphs



Remember to draw the lines on the graph to show your working.

Change £80 into Turkish lira

1) Start at 80 on the horizontal axes as this for pounds and go up vertically until you reach the line
2) From the line, read horizontally until you get to the axis showing lira

Change 600 Turkish lira to pounds

As this value is not shown by the graph, we have to use a value that is, to help.

1) Start at 200 on the vertical axes and go across horizontally until you reach the line. From the line, read vertically until you get to the axes.
2) x3 200 lira = £50 x3
600 lira = £150

Gradient

Gradient: This is the steepness of the line. The highest the number the steeper the line. We use the formula before to calculate it:

$$Gradient = \frac{difference\ in\ y}{difference\ in\ x}$$

(3, 4) and (5, 10)

Subtract the two y values.

Gradient = $\frac{10-4}{5-3} = \frac{6}{2} = 3$

Subtract the two x values.

Gradient = 3

Key Terms:

Axes: A fixed reference line on a grid to help show the position of coordinates.
Convert: Change a value or expression from one form to another.
Equation: A mathematical statement containing an equals sign.
Gradient: How steep a line is at any point.
Midpoint: The point halfway along a line or between two coordinates.
Conversion graph: A graph which converts between two variables.
Distance-time graph: A graph that shows a journey and the relationship between the distance travelled in a given time.
Real - life graph: This is a graph that represents a situation that we would see in real life.
Intercept: Where two graphs cross.
y-intercept: Where a graph crosses the y-axis.
Gradient: The rate of change of one variable with respect to another. This can be seen by the steepness.
Stationary: A person/vehicle is not moving.

You need to be able to:

- Complete and read a distance-time graph.
- Calculate speed from a graph.
- Read information from a conversion graph and use this to solve problems.
- Interpret real life graphs, including distance-time and conversion graphs.
- Calculate the gradient of a line.
- Calculate the midpoint of coordinates.
- Complete a table of values for a linear graph and draw it.

Knowledge Organiser: 6a Graphs: The Basics and Real-Life Graphs

What you need to know:

Linear graphs

Linear graphs are straight line graphs. We substitute the x value into the equation to find the y value. Once we have both, we can then plot the coordinates and draw the graph.

Draw the graph of y = 2x – 1.

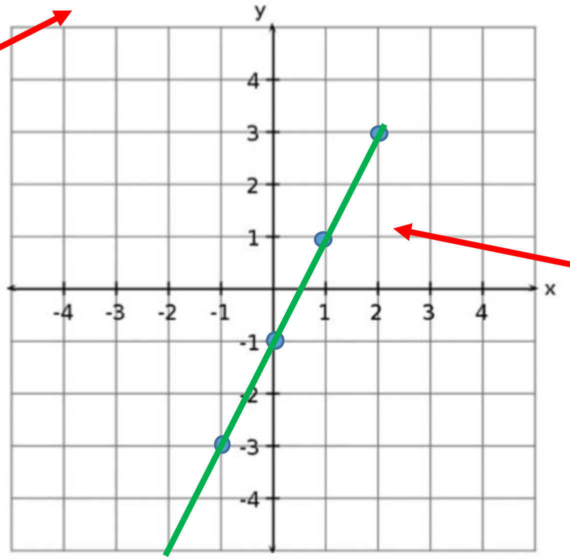
To do this we multiply the x value by 2 and then subtract 1 to get the y value.

y = 2x – 1

X	- 2	- 1	0	1	2
Y	-5	-3	-1	1	3

Multiply this value by 2 and then subtract 1 to get the y value.

This coordinate would be (-2,-5).



Don't forget to draw a straight line through all of the coordinates you have plotted.

Notice this graph has a gradient of 2 (the y values go up by 2 each time) and a y-intercept of -1 (the graph cuts through the y axis at -1).

Calculating the midpoint

Calculate the midpoint of: (3, 5) and (9, 11)

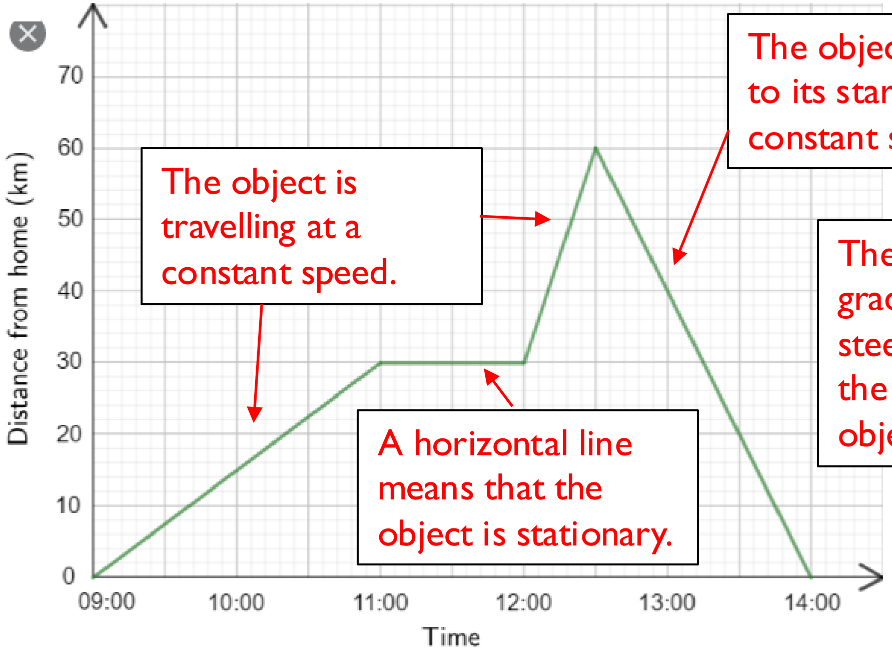
(3, 5)
+ (9, 11)
=(12,16)

Add the x values together then add the y values together.

(12,16) ÷ 2 = (6,8)

Divide both the x and y values by 2 to find the middle point.

Distance-time Graphs



The object is travelling at a constant speed.

A horizontal line means that the object is stationary.

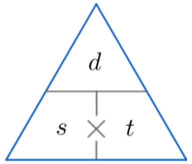
The object is returning to its starting point at a constant speed.

The greater the gradient (the steeper the line) the faster the object is moving.

The speed of an object can be calculated from the gradient of the graph.

E.g. calculate the speed at which the object travelled between 9am and 11am.

Speed = 30 ÷ 2
= 15 km/h



Knowledge Organiser: 6b Linear Graphs and Coordinate Geometry

What you need to know:

Plotting Straight Line Graphs

To plot a straight line graph, you may be given a table or you may need to draw one.

Example: Plot the graph of y = 4x – 2 for the values of x from -3 to 3.

1) Draw a table of values if you have not been given one.

x	-3	-2	-1	0	1	2	3
y							

2) Substitute in your x values to y = 4x – 2, this will give the corresponding y values.

x	-3	-2	-1	0	1	2	3
y	-14	-10	-6	-2	2	6	10

3) Plot the points on the graph and join with a ruler.

E.g. (-3, -14), (-2, -10), (-1, -6), (0, -2), ... etc

Identifying the gradient and intercept

The equations of all straight lines can be written in the form:

$$y = mx + c$$

Gradient – The number in front of the x.
This tells us how steep the line is.

Intercept – The number on its own.
Shows where the line cuts the y axis.

Example: Find the gradient and intercept of the following lines.

1)	$y = 5x - 2$		Grad = 5	Intercept = - 2
2)	$2y = 4x + 5$	$y = 2x + 2.5$	Grad = 2	Intercept = 2.5
3)	$x + y = 10$	$y = -x + 10$	Grad = - 1	Intercept = 10

Rearrange all equations so they are in the form y = mx + c (the y must be isolated)

Key Terms:

Axes: A fixed reference line on a grid to help show the position of coordinates.

Gradient: How steep a graph is at any point.

Y Intercept: Where the graph cuts through the y axis.

Perpendicular: A line that is at 90° to another line. They meet or cross at a right angle.

Parallel: Lines that are the same distance apart. They never cross.

Equation: A mathematical statement containing an equals sign.

Substitute: When a letter is replaced by a number.

Reciprocal: This is 1 divided by the given number.

You need to be able to:

- Plot and draw linear graphs from equations and tables of values.
- Identify and interpret the gradient and y intercept of a linear graph in the form y=mx + c.
- Find the equation of a line given its gradient or points it passes through.
- Interpret information presented in a range of linear graphs.
- Find the equations of parallel and perpendicular lines from coordinates and other equations.

Remember:

y = b is a horizontal line which crosses the y axis at b

x = c is a vertical line which crosses the x axis at c

Knowledge Organiser: 6b Linear Graphs and Coordinate Geometry

What you need to know:

Calculating the gradient from two points

Calculate the gradient of a line that passes through the points (4,10) and (-3,-11).

Use the formula $\frac{y_2-y_1}{x_2-x_1}$ or $\frac{Change\ in\ y}{Change\ in\ x}$

1) Label your coordinates.

(4,10) and (-3,-11).

x_1, y_1 $\quad$ x_2, y_2

2) Substitute into the formula or your choice.

$\frac{-11-10}{-3-4}$

3) Simplify the fraction.

$\frac{-21}{-7} = 3$

So the gradient of the line joining these two points is **3**.

Parallel and Perpendicular Lines

Parallel lines: The gradient of parallel lines is the same, this is why they never meet.

y = 2x + 1 $\quad$ y = 2x - 4 $\quad$ y = 2x

The gradients are all 2 here so they are all parallel.

If we are told that we want a line parallel to y = 4x + 6 and passing through the point (1, 3) then we know that the gradient of our new line is 4 so y = 4x + C. We would then substitute in (1, 3) to calculate the value of C as seen above.

Finding the equation of a line from two points

Find the equation of the line passing through the points (3,1) and (-2,-9).

1) Find the gradient, using the formula.

$$\frac{y_2-y_1}{x_2-x_1} = \frac{-9-1}{-2-3} = \frac{-10}{-5} = 2$$

2) Write out the equation replacing *m* with the found gradient.

$$y = 2x + c$$

3) Substitute in one pair of coordinates and rearrange to calculate the value of c.

$$1 = (2 \times 3) + c$$
$$1 = 6 + c$$
$$-5 = c$$

3) Re-write your equation in the form $y = mx + c$ with your calculated values of m and c.

$$\boldsymbol{y = 2x - 5}$$

Perpendicular lines: The gradient of perpendicular lines is the negative reciprocal; this is why they meet at right angles.

y = 2x $\quad$ y = -$\frac{1}{2}$x

The negative reciprocal of 2 is -$\frac{1}{2}$.

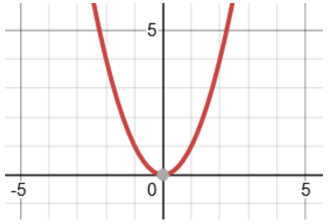
If the gradient of a line was -$\frac{2}{3}$ then the line perpendicular would have a gradient of $\frac{3}{2}$. You could then find the full equation by substituting in a given coordinate using the same method as above.

Knowledge Organiser: 6c
Quadratic Graphs

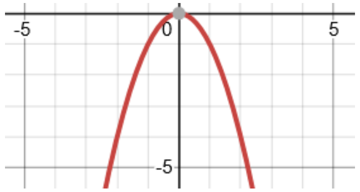
Key Concepts

A quadratic graph will always be in the shape of a parabola.

$y = x^2$

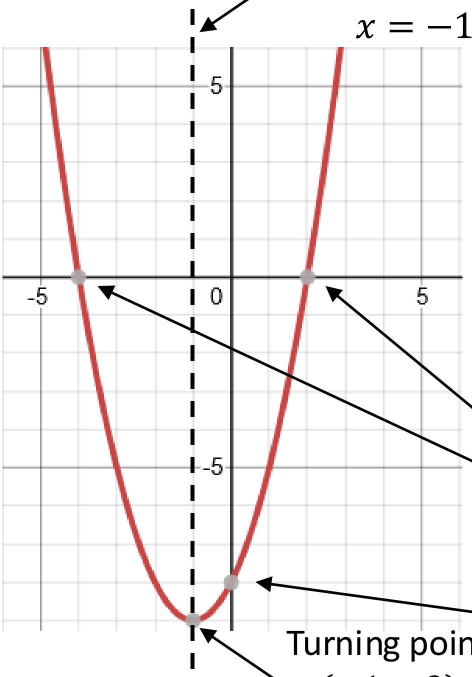


$y = -x^2$



The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.

Examples



Line of symmetry $x = -1$

Turning point $(-1, -9)$

$$y = x^2 + 2x - 8$$

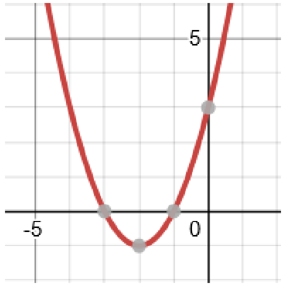
A quadratic equation can be solved from its graph.
The roots of the graph tell us the possible solutions for the equation.
There can be 1 root, 2 roots or no roots for a quadratic equation. This is dependant on how many times the graph crosses the x axis.

Roots $x = -4$
$x = 2$

y intercept = -8

Key Words

Quadratic
Roots
Intercept
Turning point
Line of symmetry



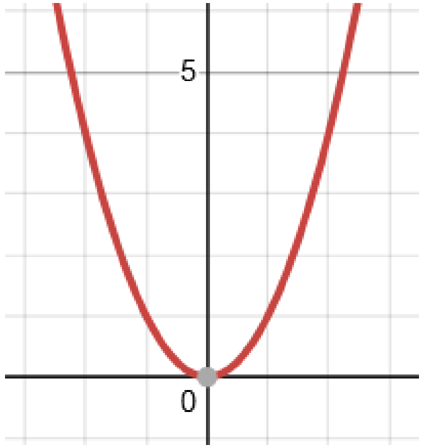
Identify from the graph of $y = x^2 + 4x + 3$:

1) The line of symmetry
2) The turning point
3) The y intercept
4) The two roots of the equation

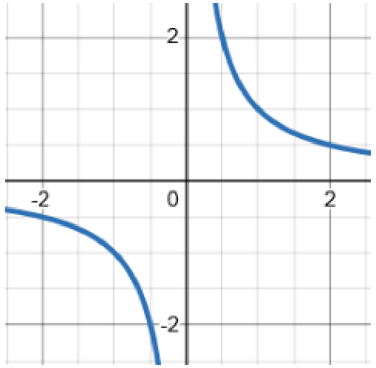
ANSWERS 1) $x = -2$ 2) $(-2, -1)$ 3) 3 4) $x = -1$ *and* $x = -3$

Knowledge Organiser: 6c
Types of Graph

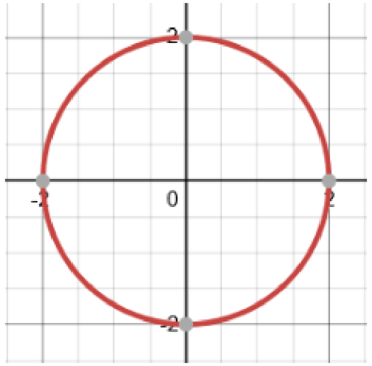
Examples



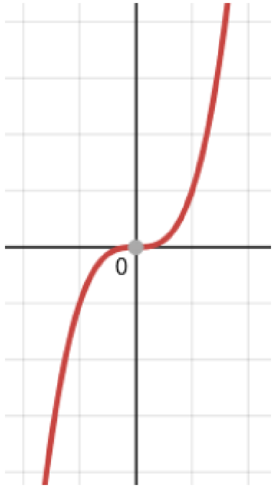
Quadratic graphs
$y = x^2$



Reciprocal graphs
$y = \frac{1}{x}$



Circle graphs
$x^2 + y^2 = 4$

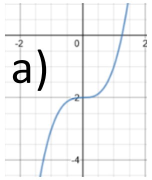


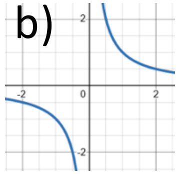
Cubic graphs
$y = x^3$

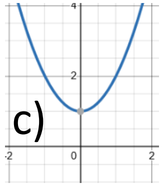
Key Words

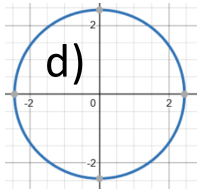
Quadratic
Cubic
Reciprocal
Circle
Graph

Match the graph with the correct equation:

a)


b)


c)


d)


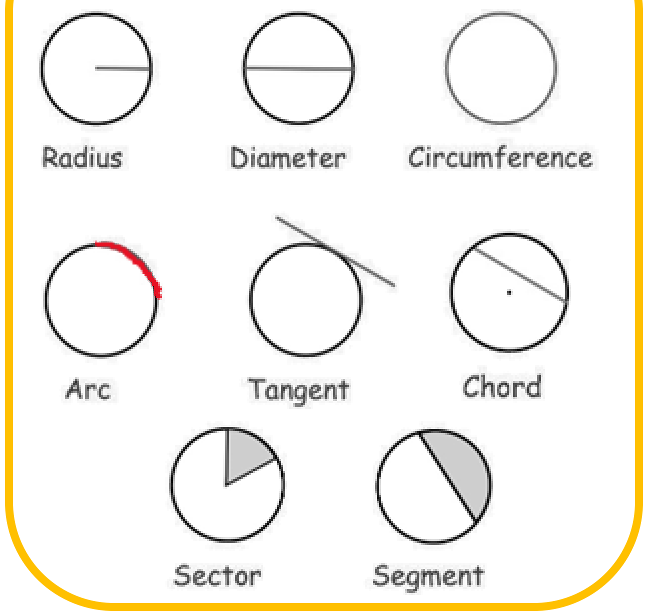
1) $x^2 + y^2 = 6$
2) $y = \frac{1}{x}$
3) $y = x^3 - 2$
4) $y = x^2 + 1$

ANSWERS: 1d 2b 3a 4c

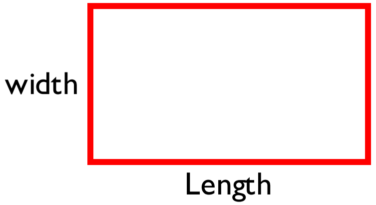
Knowledge Organiser: 7a Perimeter, Area and Circles

What you need to know:

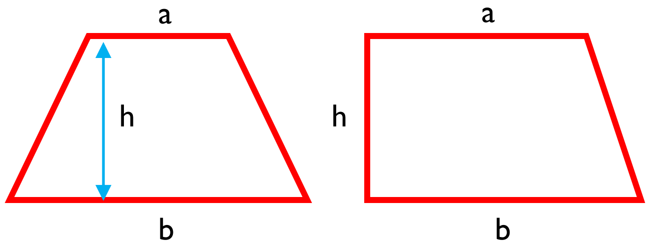
Parts of a circle



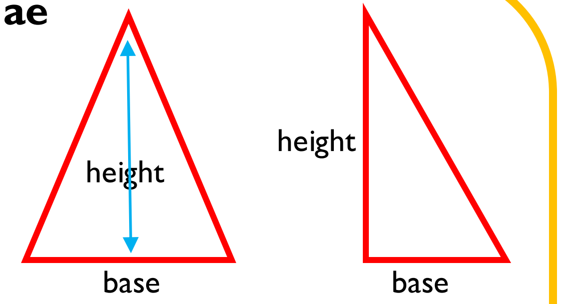
Area Formulae



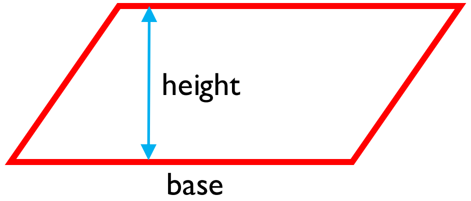
Area = Length x width



Area = ½(a + b)h



Area = $\frac{\text{base x height}}{2}$



Area = base x height

Key Terms

Area: The amount of space a surface takes up. The amount of space inside a shape.

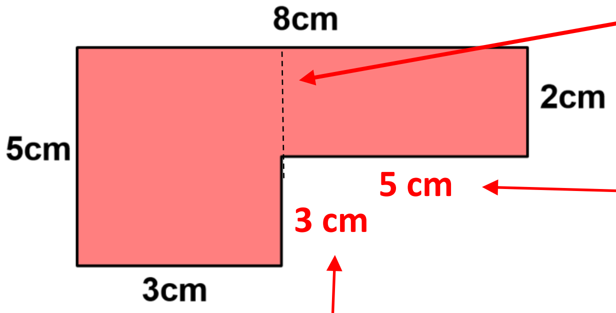
Perimeter: The distance around the outside of the shape.

Formula: A mathematical rule written using symbols, usually in the form of an equation.

Compound / composite: A shape made up of multiple shapes.

Pi: The ratio of the circumference of a circle to its diameter. 3.141592653589…

Perimeter and Area of Compound Shapes



Split the shape into shapes that you can find the area of. Remember to use the correct formulae from above.

Look at the horizontal lengths
8 – 3 = 5 cm

Look at the vertical lengths
5 – 2 = 3 cm

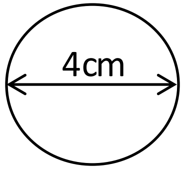
Area = (5 x 3) + (2 x 5)
= 15 + 10 = **25 cm²**

Perimeter = 3 + 5 + 8 + 2 + 5 + 3
= **26 cm**

Knowledge Organiser: 7a Tables, Charts and Graphs

Area and Circumference of Circles

Circumference = πd

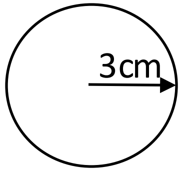


C = $\pi \times 4$

= 4π

or = 12.57cm

Area = πr^2

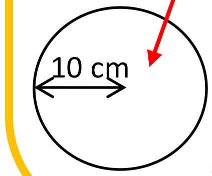


A = $\pi \times 3^2$

= 9π

or = 28.3cm^2

You have the radius, so will need to find the diameter.



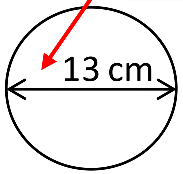
C = $\pi \times d$

= π x 20

= 20 π

Or 62.8318.. cm

You have the diameter, so first find the radius.



A = $\pi \times r^2$

= $\pi \times 6.5^2$

$= \frac{169}{4}\pi$

or 132.73228.. cm^2

Area of a Sector

To find the area of a sector we use the formula below. This helps us to calculate a fraction of the whole circle's area.

$$\text{Area} = \frac{\theta}{360} \times \pi \times r^2$$

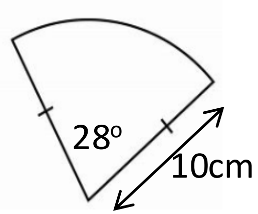
θ = theta
This represents an unknown angle.

$$\text{Area} = \frac{28}{360} \times \pi \times 10^2$$

$$\text{Area} = \frac{28}{360} \times \pi \times 100$$

$$\text{Area} = \frac{70}{9}\pi$$

or = 24.43460953 cm^2



Either leave your answer in terms of pi or as an **unrounded** decimal.

Arc length and perimeter of a sector

To find the arc length we use the formula below. The helps us to calculate a fraction of the whole circumference.

Arc Length

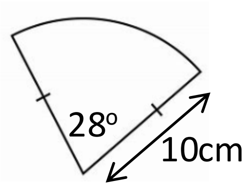
$$\text{Arc} = \frac{\theta}{360} \times \pi \times d$$

$$\text{Arc} = \frac{28}{360} \times \pi \times 2 \times 10$$

$$\text{Arc} = \frac{28}{360} \times \pi \times 20$$

$$\text{Arc} = \frac{14}{9}\pi$$

or = 4.89cm



Perimeter of sector

To find the perimeter of the sector.

1) Find the arc length
$\frac{14}{9}\pi$

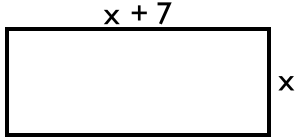
2) Add on the two straight sides (radii)
$\frac{14}{9}\pi + 10 + 10$
= 24.88692191 cm

Forming equations for area and perimeter

Sometimes the area or perimeter problem you are working on has unknown values you are trying to find. Try to form an equation.

E.g. A rectangular field is 7 metres longer than wide. The perimeter of the field is 106m. Find the length and width of the field.

1) Draw a diagram representing the problem.
2) Label the sides using algebra.
3) Form an equation.
4) Solve the equation.



ANSWER

Perimeter = x + x + 7 + x + x + 7
= 4x + 14

4x + 14 = 106 m
(- 14) (- 14)
4x = 92
x = 23 m
Length = 30 m Width = 23 m

Knowledge Organiser: 7b 3D Forms and Volume, Cylinders, Cones and Spheres

What you need to know:

Prisms: Volume = area of the cross-section x depth

Surface area: Find the area of each face and add them altogether. Use the net to help you.

The net of a 3D shape is the pattern you would get if you opened the shape up.

Volume

Area of cross-section (triangle) = $\frac{4\times6}{2} = 12cm^2$

Volume = $12 \times 12 = \mathbf{144cm^3}$

Surface area

Area a = 7 x 12 = 84cm²

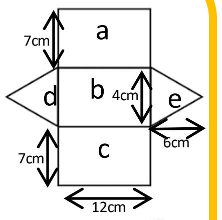
Area b = 4 x 12 = 48cm²

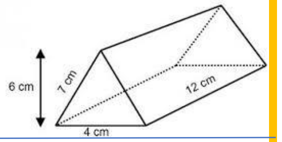
Area c = 7 x 12 = 84cm²

Area d = $\frac{4\times6}{2} = 12cm^2$

Area e = $\frac{4\times6}{2} = 12cm^2$

Total surface area = 84+48+84+12+12 = **240cm²**





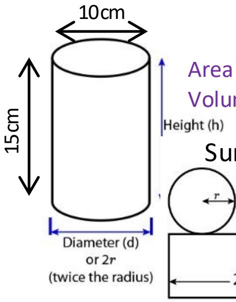
Cylinders: Volume = area of the cross-section (circle) x depth

$= \pi r^2 \times \text{h}$

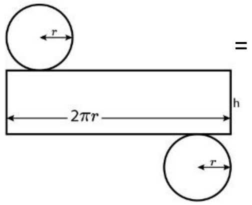
Area of cross-section (circle) $= \pi \times 5^2 = 25\pi$

Volume = $25\pi \times 15 = 375\pi cm^3$

$= 1178.1cm^3$



Surface area = sum of the areas of the faces.



$= 2\pi r^2 + \pi\text{dh}$

Surface area $= 2 \times \pi \times 5^2 + \pi \times 10 \times 15$

$= 50\pi + 150\pi$

$= 200\pi cm^2$

$= 638.32cm^2$

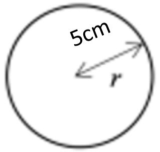
Spheres: Volume = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 5^3$

$= \frac{500}{3}\pi$

$= 166.\dot{6}\pi$

$= 523.6cm^3$



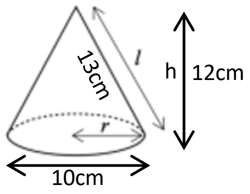
Surface area = $4\pi r^2$

$= 4\pi \times 5^2$

$= 100\pi$

$= 314.16cm^2$

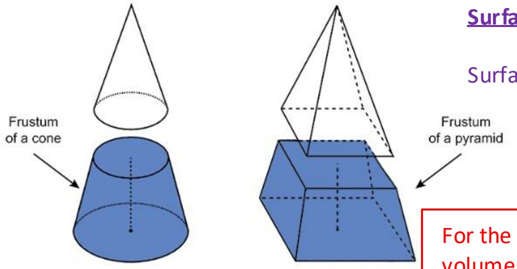
Cones and Pyramids: Volume = $\frac{area\ of\ the\ base\ x\ vertical\ height}{3}$



Surface area of a cone = $\pi r^2 + \pi\text{rl}$

You may have to use Pythagoras' theorem to find the slant height (L)

L = $\sqrt{r^2 + h^2}$



Volume

Area of base (circle) $= \pi \times 5^2 = 25\pi$

Volume = $\frac{25\pi\times 12}{3} = 100\pi$

$= 314.16cm^3$

Surface area

Surface area = $\pi r^2 \times \pi\text{rl}$ L = $\sqrt{5^2 + 12^2} = 13cm$

$= \pi \times 5^2 + \pi \times 5 \times 13$

$= 25\pi + 65\pi$

$= 90\pi cm^2$

$= 282.74cm^2$

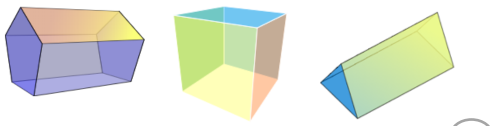
For the volume of a frustum, first calculate the volume of the large cone/pyramid, then the volume of the smaller cone/pyramid and subtract.

Key Terms:

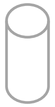
Surface Area The total area of the surface of a 3D object. Units: cm^2, m^2 etc.

Volume The amount of 3-dimensional space something takes up. Units: cm^3, m^3 etc.

Prism A solid object with a constant cross-section and flat faces.



Cylinder A solid 3D object with a constant circular cross-section.

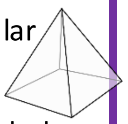


Sphere A 3D object shaped like a ball.

Hemisphere Exactly half a sphere.

Cone A solid 3D object that has a circular base and joins to an apex.

Pyramid A solid object where the base is a polygon and all the triangular sides meet at a single vertex.



Frustum A pyramid or cone with the top cut off flat.

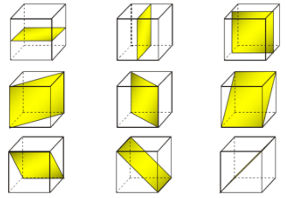
Cross-section The shape we get when cutting straight through an object.

Key Facts:

Volume can also be called Capacity.

1cm³ = 1ml

Planes of symmetry – a plane of symmetry divides a three-dimensional shape into two halves that are mirror images of each other.



A cube has 9 planes of symmetry.

Knowledge Organiser: 7c Accuracy and Bounds

What you need to know: Rounding and Truncation to state error intervals

The upper and lower bound come from the largest and smallest values that would **round** to a particular number.

Take 'half a unit above and half a unit below'. For example rounded to 1 d.p means nearest 0.1, so add 0.05 and subtract 0.05 to get the bounds.

All error intervals look the same like this:

$$\leq x <$$

The lowest value a number could have been is the lower bound.
The highest value a number could have been is the upper bound.

E.g. 1 State the upper and lower bound of 360 when it has been **rounded** to 2 significant figures:
2 significant figures is the nearest 10, so 'half this' to get 5, and add on to 360 and take it off 360,

$$355 \leq x < 365$$

Note: You should know it could be 364.9999… but we write 365 as the upper bound for ease of calculations, and use the < sign to show that x must be smaller than 365.

e.g. 2 **Truncation:** State the error interval of 4.5 when it has been truncated to 1 decimal place. Truncated means that the trailing digits have been "chopped off" not rounded. The lowest value it could have been is 4.5 and the highest is 4.5999999... so it must be smaller than 4.6. **The error interval is : $4.5 < x \leq 4.6$**

Key Terms:

- Bound
- Upper
- Lower
- Accuracy
- Rounding
- 'to the nearest'
- Truncation
- Suitable degree of accuracy is based on an agreement of UB & LB to the most sig. fig.

Key Facts:

Rounding a number and **truncating** are different things.

Truncation comes from the word *truncare*, meaning "to shorten," and can be traced back to the Latin word for the trunk of a tree, which is *truncus*.

3.14159265… can be truncated to 3.1415 (note that if it had been rounded, it would become 3.1416).

A question may ask for the error interval for **rounding** or **truncation** – take care to read the question!

Knowledge Organiser: 7c Accuracy and Bounds

What you need to know: Using Upper and Lower Bounds in calculations

When completing calculations involving boundaries we are aiming to find the greatest or smallest answer. The table shows how to find these in each operation. If combined, e.g. an add then divide, do each operation separately

	+	-	×	÷
Upper bound answer	$UB_1 + UB_2$	$UB_1 - LB_2$	$UB_1 \times UB_2$	$UB_1 \div LB_2$
Lower bound answer	$LB_1 + LB_2$	$LB_1 - UB_2$	$LB_1 \times LB_2$	$LB_1 \div UB_2$

A restaurant provides a cuboid stick of butter to each table. The dimensions are 30mm by 30mm by 80mm, correct to the nearest 5mm. Calculate the upper and lower bounds of the volume of the butter.

$$Volume = l \times w \times h$$

$$Upper\ bound = 32.5 \times 82.5 \times 32.5$$
$$= 87140.63mm^3$$

$$Lower\ bound = 27.5 \times 77.5 \times 27.5$$
$$= 58609.38mm^3$$

$$D = \frac{x}{y}$$

x = 99.7 correct to 1 decimal place.
y = 67 correct to 2 significant figures.
Give the value of D to a **suitable degree of accuracy.**

Find the Upper and lower bound first.

$$Upper\ bound\ D = \frac{99.75}{66.5} = 1.5$$

$$Lower\ bound\ D = \frac{99.65}{67.5} = 1.48$$

Then decide what the number would round to, to be the same number. To 2 d.p. they would be different. To 1.d.p. they would both be 1.5.
The answer is therefore 1.5 correct to 1 decimal place as your answer.

Key Terms:

- Bound
- Upper
- Lower
- Accuracy
- Rounding
- Suitable Degree of Accuracy

Key Facts:

The boundaries of a number derive from **rounding**. Do this before completing the calculation (you get marks!)

E.g. State the boundaries of 360 when it has been rounded to 2 significant figures:

$$355 \leq x < 365$$

E.g. State the boundaries of 4.5 when it has been rounded to 2 decimal place:

$$4.45 \leq x < 4.55$$

These boundaries can also be called the **error interval** of a number.

Knowledge Organiser: 8a
Reflection, Rotation and Translation

Key Concepts

A **reflection** creates a mirror image of a shape on a coordinate graph. The mirror line is given by an equation eg. $y = 2, x = 2, y = x$. The shape does not change in size.

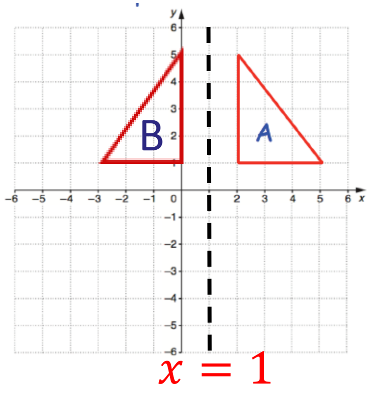
A **rotation** turns a shape on a coordinate grid from a given point. The shape does not change size but does change orientation.

A **translation** moves a shape on a coordinate grid. Vectors are used to instruct the movement:

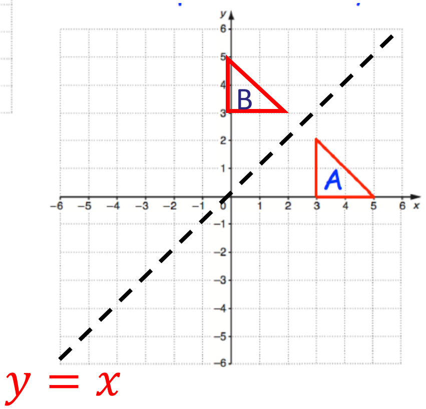
$\begin{pmatrix} x \\ y \end{pmatrix}$ → Positive-Right, Negative - Left; → Positive-Up, Negative - Down

Examples

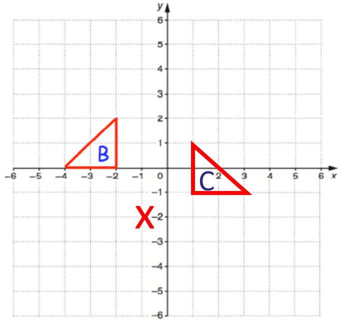
Reflect shape A in the line $x = 1$. Label it B.



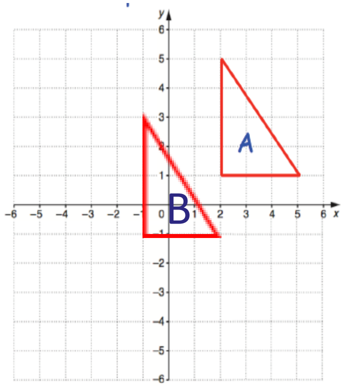
Reflect shape A in the line $y = x$. Label it B.



Rotate shape B from the point (-1, -2)



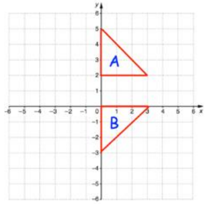
Translate shape A by $\binom{-3}{-2}$. Label it B

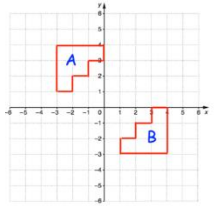


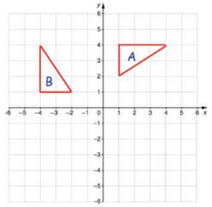
Key Words

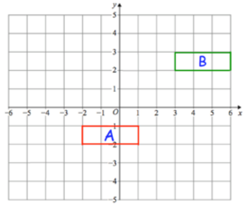
Rotate
Clockwise
Anticlockwise
Centre
Degrees
Reflect
Mirror image
Translate
Vector

Describe the **single** transformation you see on each coordinate grid from A to B:









ANSWERS: a) reflection, $y = 1$ b) reflection $y = x$ c) rotation, centre (0,0), 90° anticlockwise
d) translation $\binom{5}{4}$

Knowledge Organiser : 8a
Enlargement

Key Concepts

An **enlargement** changes the size of an image using a scale factor from a given point.

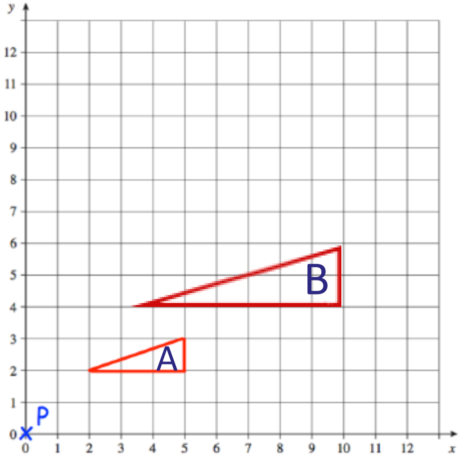
A **positive scale factor** will increase the size of an image.

A **fractional scale factor** will reduce the size of an image.

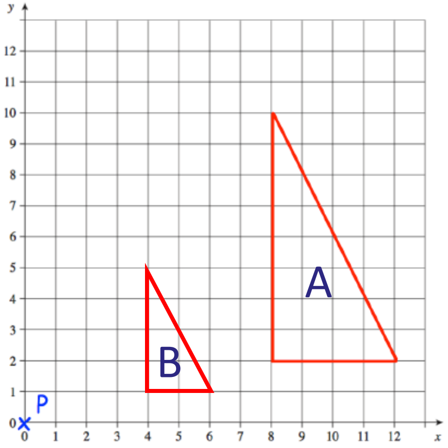
A **negative scale factor** will place the image on the opposite side of the centre of enlargement, with the image inverted.

Examples

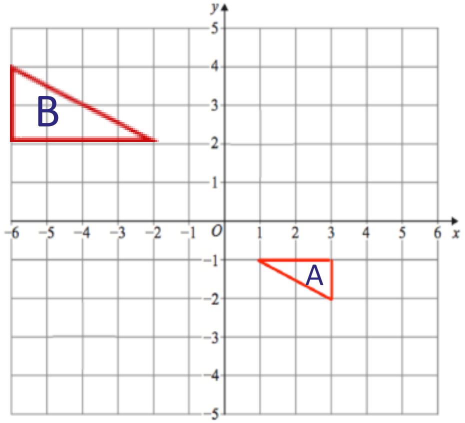
Enlarge shape A by scale factor 2 from point P.



Enlarge by scale factor $\frac{1}{2}$ from point P.

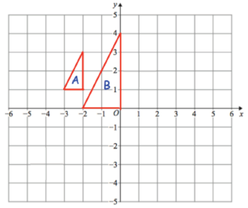


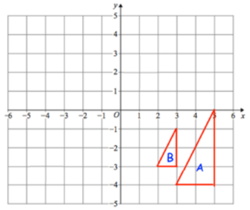
Enlarge by scale factor -2 from (0,0).

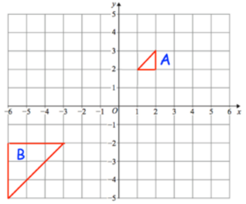


Key Words
Enlargement
Scale factor
Centre
Positive
Negative

Describe the **single** transformation you see on each coordinate grid from A to B:







ANSWERS: a) enlarge, centre (-4,2) scale factor 2   b) enlarge, centre (1,-2) scale factor $\frac{1}{2}$

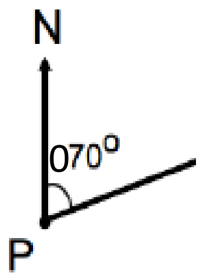
c) enlarge, centre (0,1) scale factor -3

Knowledge Organiser: 8b
Scales and Bearings

Key Concepts

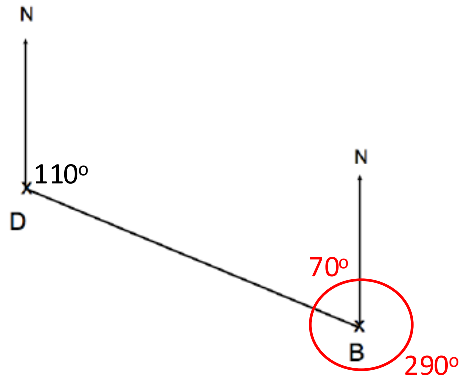
Scales are used to reduce real world dimensions to a useable size.

A **bearing** is an angle, measured **clockwise** from the **north** direction. It is given as a **3 digit** number.



Examples

The diagram shows the position of a boat B and dock D.



The scale of the diagram is 1cm to 5km.

a) Calculate the real distance between the boat and the dock.

$$6cm = 6 \times 5$$
$$= 30km$$

b) State the bearing of the boat from the dock.

110^o

c) Calculate the bearing of the dock from the dock.

$180^o - 110^o = 70^o$ because the angles are cointerior

$360^o - 70^o = 290^o$ because angles around a point equal 360°

Key Words

Scale

Bearing

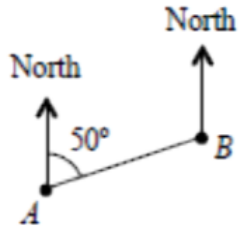
Clockwise

North

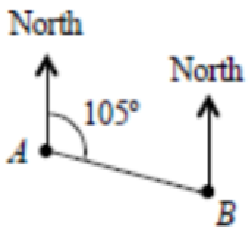
Find the bearing of A from B

(Diagrams not drawn to scale):

a)



b)

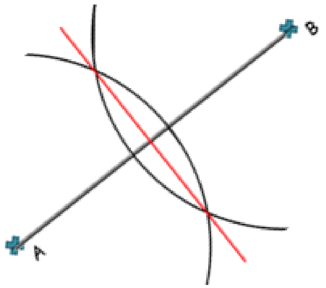


ANSWERS: (a) 230o (b) 285o

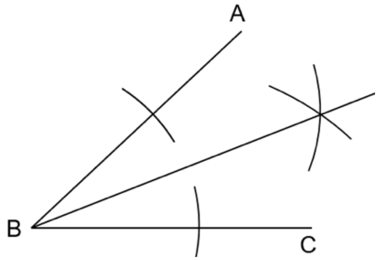
Knowledge Organiser: 8b
Constructions and Loci

Key Concepts

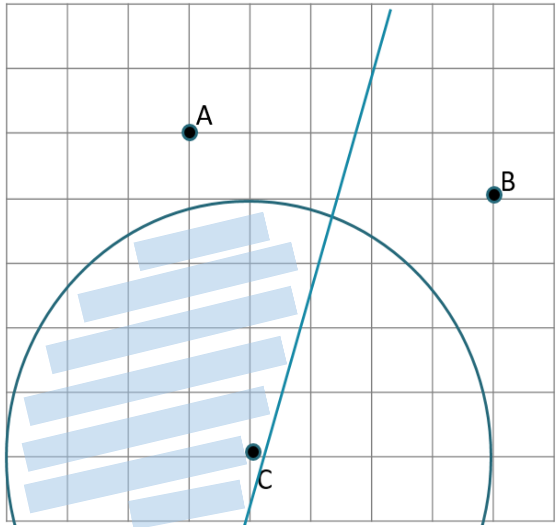
Line bisector



Angle bisector



Examples

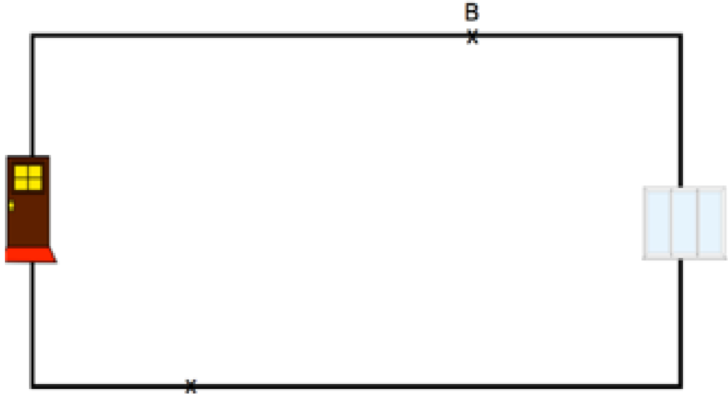


Shade the region that is:

- closer to A than B
- less than 4 cm from C

Line bisector of A and B

Circle with radius 4cm



1cm = 1m

There are two burglar alarm sensors, one at A and one at B.

The range of each sensor is 4m.

The alarm is switched on.

Is it possible to walk from the front door to the patio door without setting off the alarm?

Key Words

Bisect

Radius

Region

Shade

Knowledge Organiser: 9a Solving Quadratic and Simultaneous Equations

What you need to know:

Solving quadratics by completing the square

$For\ x^2 + bx + c = 0$

$$\left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2 = 0$$

Solve $x^2 + 6x + 5 = 0$

Add half the coefficient of the x term, put this in a bracket and square the expression

$$\left(x + \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 5 = 0$$

Subtract the square of this number

Simplify this as much as possible:

$$(x + 3)^2 - 3^2 + 5 = 0$$

$$(x + 3)^2 - 4 = 0$$

Solve the equation

$Either$: $x = \sqrt{4} - 3 = 1$

Or: $x = -\sqrt{4} - 3 = -5$

Reminders:

_ x _ = -3
_ + _ = -2

Factorise a quadratic:

$$x^2 - 2x - 3 = (x - 3)(x + 1)$$

Factorise and solve a quadratic:

$$x^2 + 4x - 5 = 0$$

$$(x - 1)(x + 5) = 0$$

Therefore the solutions are:

Either $(x - 1) = 0$

$x = 1$

Or $(x + 5) = 0$

$x = -5$

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve $x^2 + 4x - 2 = 0$ (to 2 d.p.)

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 + 8}}{2}$$

$x = 0.45$ Or $x = -4.45$

Key Terms:

Factorise: Putting an expression back into brackets

Solve: Find the values (or values) which can be put into an equation to make it true

Quadratic Equations: Equations which involve the second power of a variable e.g. x^2 or y^2

Simultaneous Equations: More than one equation which involve more than one variable. The variables have the same value in each equation

Completing the Square: Rewriting the expression so it becomes a complete square and can be solved

You need to be able to:

- Factorise quadratic expressions
- Solve quadratic equations by factorisation, completing the square and by using the quadratic formula
- Find the exact solutions of two simultaneous equations with two unknowns linear / quadratic
- Set up and solve a pair of simultaneous equations in two variables to represent a situation

Knowledge Organiser: 9a Solving Quadratic and Simultaneous Equations

What you need to know:

Solving Simultaneous Equations

Two linear equations:

$$\begin{aligned} 3x + 2y &= 18 \\ 3x - y &= 9 \end{aligned}$$

$\times 2$

$$\begin{aligned} 3x + 2y &= 18 \\ 6x - 2y &= 18 \end{aligned}$$

$+$

SSS – Same Sign Subtract
DSA – Different Sign Add

$$9x = 36$$

$$\underline{x = 4}$$

Substitute in $x = 4$ into an original equation

$$3x + 2y = 18$$

$$(3 \times 4) + 2y = 18$$

$$12 + 2y = 18$$

$$2y = 6$$

$$\underline{y = 3}$$

Substitute both values into the other equation to check your solution

One linear and one quadratic equation:

$$x^2 + y^2 = 17$$

$$y = x - 3$$

Substitute $y = x - 3$ into y into the quadratic equation.

$$x^2 + (x - 3)^2 = 17$$

$$x^2 + x^2 - 6x + 9 - 17 = 0$$

$$2x^2 - 6x - 8 = 0$$

Solve by factorising or using the quadratic formula.

$$x = 4 \text{ or } x = -1$$

Substitute the x values into the linear equation to find the corresponding y values.

$$When\ x = 4, \qquad y = 4 - 3 = 1$$

$$When\ x = -1, \qquad y = -1 - 3 = -4$$

Solve simultaneous equations from worded problems:

Mr and Mrs Smith take their two children to the cinema. The total cost is £33. Mr Jones takes his three children to the cinema and the total cost is £27.50. Calculate the price of a child's ticket and an adult's ticket.

Let a be the cost of an adult ticket and c the cost of a child's ticket.

$$\begin{aligned} 2a + 2c &= 33 \\ a + 3c &= 27.5 \end{aligned}$$

$\times 2$

$$\begin{aligned} 2a + 6c &= 55 \\ - \quad 2a + 2c &= 33 \end{aligned}$$

$$4c = 22$$

$$\underline{c = 5.5}$$

Substitute c = 5.5 into the original equation:

$$2a + 11 = 33$$

$$2a = 22$$

$$\underline{a = 11}$$

A child's ticket costs £5.50 and an adult's ticket costs £11.

Knowledge Organiser: 9b Inequalities

What you need to know:

Representing Inequalities on a Number Line

On a **number line** we use circles to highlight the key values:

○ is used for less/greater than

● is used for less/greater than or equal to

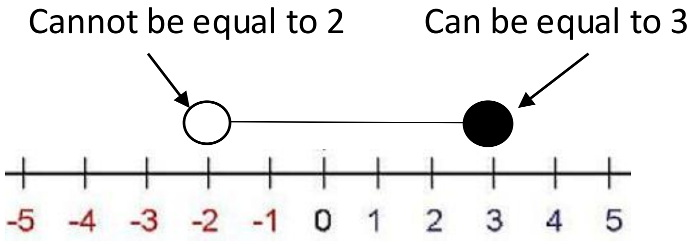
State the values of n that satisfy:

$$-2 < n \leq 3$$

Cannot be equal to 2 Can be equal to 3

-1, 0, 1, 2, 3

b) Show this inequality on a number line:



Reminders:

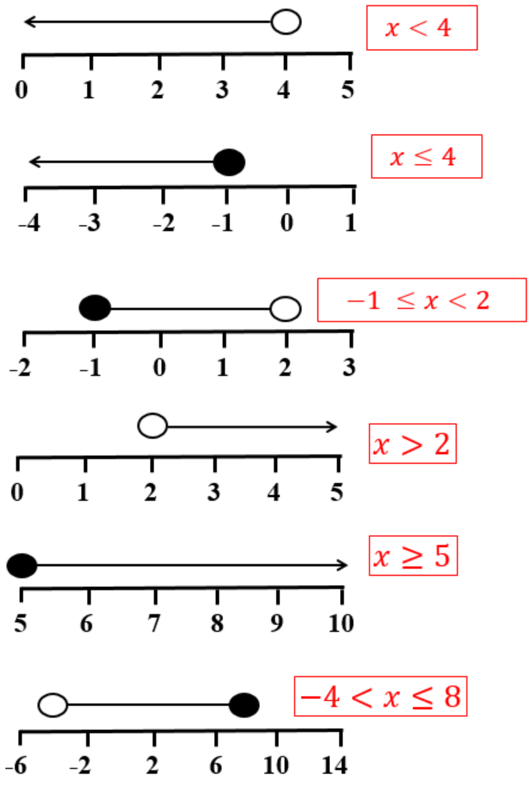
$x < 2$ means x is less than 2

$x \leq 2$ means x is less than or equal to 2

$x > 2$ means x is greater than 2

$x \geq 2$ means x is greater than or equal to 2

Representing Inequalities on a Number Line



Key Terms:

Inequality: The relationship between two expressions that are not equal

Integer: A whole number

Solve: Find a numerical value that satisfies the equation or inequality

Inverse operation: The operation that reverses the effect of another operation e.g. subtraction in the inverse of addition

You need to be able to:

- Show inequalities on number lines
- Write down whole number values that satisfy an inequality
- Solve simple linear inequalities in one variable, and represent the solution set on a number line
- Solve two linear inequalities in x, find the solution sets and compare them to see which value of x satisfies both
- Solve linear inequalities in two variables algebraically
- Use the correct notation to show inclusive and exclusive inequalities.

Knowledge Organiser: 9b Inequalities

What you need to know:

Solving Inequalities (one unknown)

Solve this inequality and represent your answer on a **number line:**

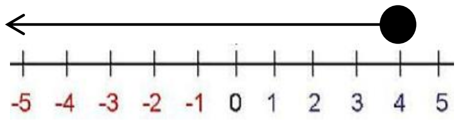
$5x - 6 \leq 14$

$+6 \quad +6$

$5x \leq 20$

$\div 5 \quad \div 5$

$x \leq 4$



Solve the inequality with the same steps as solving an equation

Solve this inequality and represent your answer on a **number line:**

When there are two inequality signs, you must solve both sides of the inequality to give a range of answers

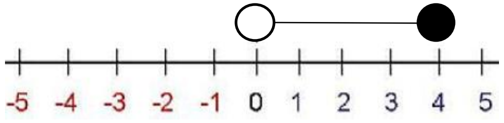
$4 < 3x + 1 \leq 13$

$-1 \qquad -1$

$3 < 3x \leq 12$

$\div 3 \quad \div 3$

$1 < x \leq 4$



Possible values of x: 2, 3, 4

Solving Inequalities (unknowns on both sides)

Solve this inequality and represent your answer on a **number line:**

$3(x - 2) \leq 14 - x$

Expand the bracket

$3x - 6 \leq 14 - x$

$+x \qquad +x$

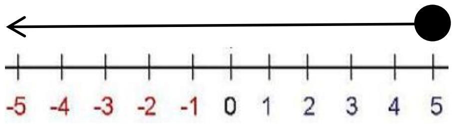
$4x - 6 \leq 14$

$+6 \quad +6$

$4x \leq 20$

$\div 4 \quad \div 4$

$x \leq 5$



Work out the integer values of x that satisfy both inequalities:

$3x - 4 \leq 11$

$+4 \quad +4$

$3x \leq 15$

$\div 3 \quad \div 3$

$x \leq 5$

$2x + 3 > 9$

$-3 \quad -3$

$2x > 6$

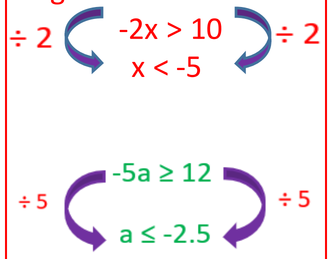
$\div 2 \quad \div 2$

$x > 3$

$x \leq 5$ and $x > 3$ so x can be 4 or 5

TOP TIP :
If you divide through an inequality with a negative number, you must reverse the inequality sign.

e.g.



$\div 2$ $-2x > 10$ $\div 2$

$x < -5$

$\div 5$ $-5a \geq 12$ $\div 5$

$a \leq -2.5$

Knowledge Organiser: 10
Venn Diagrams

Key Concepts

Venn diagrams show all possible relationships between different sets of data.

Probabilities can be derived from Venn diagrams. Specific notation is used for this:

P(A ∩ B) = Probability of A **and** B

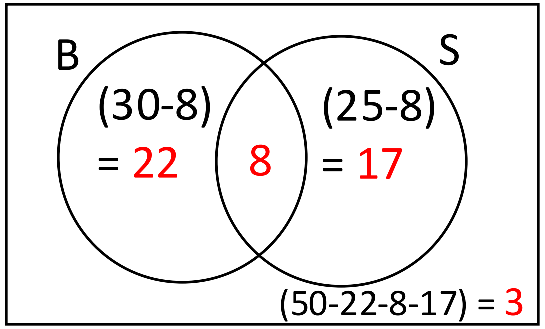
P(A ∪ B) = Probability of A **or** B

P(A') = Probability of **not** A

Example

Out of 50 people surveyed:

30 have a brother
25 have a sister
8 have both a brother and sister



a) Complete the Venn diagram

b) Calculate:

i) P(A ∩ B) $= \frac{8}{50}$ ii) P(A ∪ B) $= \frac{47}{50}$ iii) P(B') $= \frac{20}{50}$

iv) The probability that a person with a sister, does not have a brother.

$= \frac{17}{25}$

Key Words
Venn diagram
Union
Intersection
Probability
Outcomes

40 students were surveyed:

20 have visited France
15 have visited Spain
10 have visited both France and Spain

a) Complete a Venn diagram to represent this information.

b) Calculate:

i) P(F ∩ S) ii) P(F ∪ S) iii) P(S')

iv) The probability someone who has visited France, has not gone to Spain.

ANSWERS: bi) 10/40 ii) 25/40 iii) 15/40 iv) 10/20 = 1/2

Knowledge Organiser: 10
Two-Way Tables and Probability Tables

Key Concepts

Two way tables are used to tabulate a number of pieces of information.

Probabilities can be formulated easily from two way tables.

Probabilities can be written as a **fraction, decimal or a percentage** however we often work with fractions. You do not need to simplify your fractions in probabilities.

Estimating the number of times an event will occur

Probability × no. of trials

Examples

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	$3x$	x-5	$2x$

A counter is chosen at random, the probability it is red is $\frac{9}{100}$. Work out the probability is black.

$$9 + 3x + x - 5 + 2x = 100$$
$$6x + 4 = 100$$
$$x = 16$$

Number of black counters = 16 – 5

= 11

Probability of choosing black = $\frac{11}{100}$

80 children went on a school trip. They went to London or to York.

23 boys and 19 girls went to London. 14 boys went to York.

	London	**York**	**Total**
Girls	19	**24**	**43**
Boys	23	14	**37**
Total	**42**	**38**	80

What is the probability that a person is chosen that went to London? $\frac{42}{80}$

If a girl is chosen, what is the probability that she went to York? $\frac{24}{38}$

Key Words

Two way table

Probability

Fraction

Outcomes

Frequency

	1	**2**	**3**
Prob	0.37	$2x$	x

1a) Calculate the probability of choosing a 2 or a 3.

b) Estimate the number of times a 2 will be chosen if the experiment is repeated 300 times.

2a) Complete the two-way table:

	Year Group			**Total**
	9	**10**	**11**	
Boys			125	407
Girls		123		
Total	303	256		831

b) What is the probability that a Y10 is chosen, given that they are a girl .

ANSWERS: 1) a) P(2) = 0.42 P(3) = 0.21 P(2 or 3) = 0.63 b) 0.42×300=126 2b) 123/424

Knowledge Organiser: 10
Probability Tree Diagrams

Key Concepts

Independent events are events which do not affect one another.

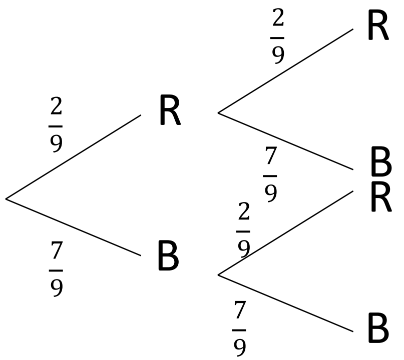
Dependent events affect one another's probabilities. This is also known as **conditional probability**.

Key Words

Independent
Dependant
Conditional
Probability
Fraction

Examples

There are red and blue counters in a bag.
The probability that a red counter is chosen is $\frac{2}{9}$.
A counter is chosen and **replaced**, then a second counter is chosen.
Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



Prob of two reds:

$$\frac{2}{9} \times \frac{2}{9} = \frac{4}{81}$$

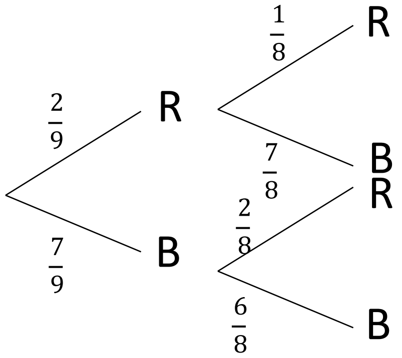
Prob of two blues :

$$\frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$$

Prob of same colours:

$$\frac{4}{81} + \frac{49}{81} = \frac{53}{81}$$

There are red and blue counters in a bag.
The probability that a red counter is chosen is $\frac{2}{9}$.
A counter is chosen and **not replaced**, then a second counter is chosen.
Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



Prob of two reds:

$$\frac{2}{9} \times \frac{1}{8} = \frac{2}{72}$$

Prob of two blues :

$$\frac{7}{9} \times \frac{6}{8} = \frac{42}{72}$$

Prob of same colours:

$$\frac{2}{72} + \frac{42}{72} = \frac{44}{72}$$

1) There are blue and green pens in a drawer.
There are 4 blues and 7 greens.
A pen is chosen and then **replaced**, then a second pen is chosen.
Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

2) There are blue and green pens in a drawer.
There are 4 blues and 7 greens.
A pen is chosen and **not replaced**, then a second pen is chosen.
Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

ANSWERS: 1) 56/121 2) 56/110

Knowledge Organiser: 11 Multiplicative Reasoning

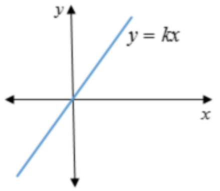
What you need to know:

Direct vs Indirect Proportion

k is the constant of proportionality

If y is directly proportional to x:

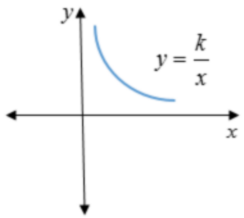
$y \propto x$ $y = kx$



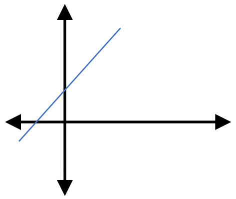
If y=mx + c, then k represents the gradient of the line y = kx

If y is indirectly proportional to x:

$y \propto \frac{1}{x}$ $y = \frac{k}{x}$



If the line does not pass through the origin, the relationship is not directly proportional.



If you are given a table of values, you can plot them to check

The height h cm of a plastic cylinder is inversely proportional to its radius r cm. A plastic cylinder of height 6cm has a radius of 4cm.
Work out the height of a cylinder with a radius of 3cm.

Step 1 – Write down the formula using k as the constant $h = \frac{k}{r}$

Step 2 – Substitute in the values of y and x $6 = \frac{k}{4}$

Step 3 – Solve the equation to find the value of k $k = 24$

Step 4 – Re-write the original equation substituting k for the actual value $h = \frac{24}{x}$

Step 5 – Substitute the new radius into this equation $h = \frac{24}{3} = 8cm$

Key Terms:

Direct Proportion: Two quantities increase at the same rate.

Indirect Proportion: As one quantity increases, the other decreases at the same rate.

Best Value: The item that is cheapest per unit/100g, e.g. the box with the lowest price per teabag

Compound Measurement: A measure made up of two or more measurements (e.g. speed, pressure, density)

More Relationships

As well as $y \propto x$ and $y \propto \frac{1}{x}$, watch out for:

y directly proportional to x^2	$y \propto x^2$	$y = kx^2$
y directly proportional to $\sqrt{x}$	$y \propto \surd$	$y = k\sqrt{x}$
y indirectly proportional to x^2	$y \propto \frac{1}{x^2}$	$y = \frac{k}{x^2}$

You need to be able to:

- Use standard units of measure, e.g. time, length, mass
- Use compound units of measure, e.g. speed, density
- Rearrange equations to solve for unknowns
- Use the equations for direct/indirect proportion to find the constant k
- Recall and use the formula for compound interest
- Change g/cm^3 to kg/m^3, kg/m^2 to g/cm^2, m/s to km/h
- Solve proportion problems using the unitary method

Knowledge Organiser: 11 Multiplicative Reasoning

What you need to know:

Best Buys

Boxes of tissues come in 3 sizes:

A) 24 Tissues = £7.99
B) 20 Tissues = £7.33
C) 15 Tissues = £5.65

Which box is the best value for money?

Step 1 – Find the price per tissue for each box

Box A = £7.99 ÷ 24
= £0.333 or 33.3p/tissue

Box B = £7.33 ÷ 20
= £0.367 or 36.7p/tissue

Box C = £5.65 ÷ 15
= £0.377 or 37.7p/tissue

Step 2 – Identify which box is cheapest per tissue

The best value for money is Box A as it is the cheapest per tissue.

Questions like these can typically be solved by first finding the amount per unit (e.g. weight per cm, price per tissue). This is called the ‘unitary method’

Indirect Proportion

It takes 7 people 10 days to paint a house. How many days would it take 5 people?

Check: does my answer make sense? Yes, it will take longer to paint the house when we have fewer people, this is an example of indirect proportion.

Step 1 – Find the time taken for 1 person
7 x 10 = 70 days

Step 2 – Divide this by the new number of people
70 ÷ 5 = 14 days

Growth and Decay

Mo invests £300 at a compound interest rate of 3% per annum. How much money is in his account after 4 years?

Step 1 – Calculate the interest rate as a decimal multiplier = 1.03

If the value was decreasing, our multiplier would be <1

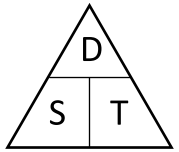
Step 2 – Substitute values into the formula for compound interest
= £300 x 1.03^4
= £337.65

$$\text{Final Amount} = \text{Starting Amount} \times \left(\text{Decimal Multiplier}\right)^n$$

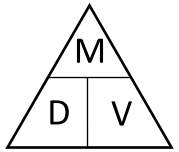
Compound Measures

Don’t forget your formula triangles.

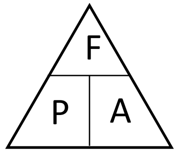
Speed:



Density:



Pressure:



Copper has a density of 8.92g/cm^3. Silver has a density of 10.49g/cm^3. 20cm^3 of copper and 5cm^3 of silver are mixed to form a new metal.

a) What is the density of the new metal?

Step 1 – Calculate the mass of each metal

Mass = Density x Volume

Copper = 8.92 x 20
= 178.4g

Silver = 10.49 x 5
= 52.45g

Step 2 – Add the masses together
Add the volumes together

178.4 + 52.45 = 230.85g
20 + 5 = 25cm^3

Step 3 – Use the formula for density using your new mass and volume

Density = M÷V
= 230.85 ÷ 25 = 9.234g/cm^3

b) Convert your answer into kg/m^3

g → kg = ÷ 1000
cm^3 → m^3 = x 1,000,000

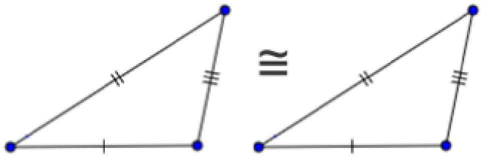
Overall, we multiply by 1000

= 9234 kg/m^3

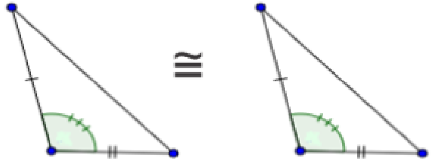
Knowledge Organiser: 12 Similarity and Congruence in 2D and 3D

What you need to know:

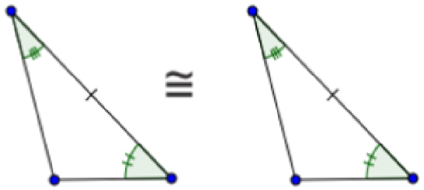
Proving Congruency of triangles



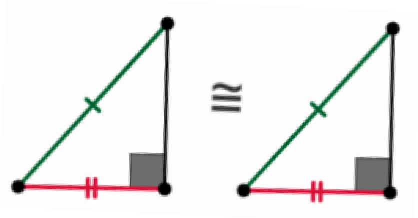
SSS = 3 sides on triangle A are equal to those on triangle B



SAS = 2 sides with the included angle on triangle A are equal to those on triangle B



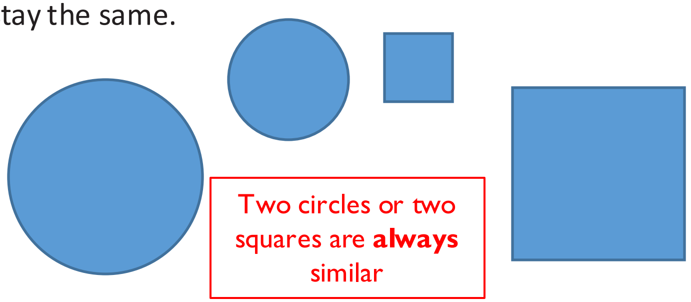
ASA = 2 angles with the included side on triangle A are equal to those on triangle B



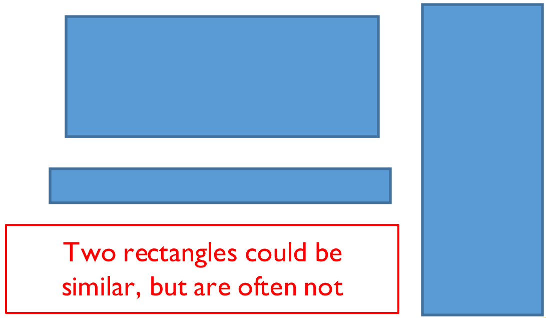
RHS = When the hypotenuse and another side on triangle A are equal to those on triangle B

Similarity

Similar figures are identical in shape, but not necessarily in size. If two shapes are similar their corresponding sides are in the same ratio and their corresponding angles are equal. When one shape is an enlargement of another, the length, area and volume change but the angles stay the same.



Two circles or two squares are **always** similar



Two rectangles could be similar, but are often not

Key Terms:

Similarity – Two shapes are Similar when one can become the other after a resize, flip, slide or turn.

Congruence –Two shapes are congruent if they are the same (shape and size) - if the lengths of the sides and the angles are the same.

Enlargement – a type of transformation that makes a shape bigger or smaller

Scale Factor – A scale factor is a number which scales, or multiplies, some quantity. For example, a scale factor of 2 means that the new shape is double the size of the original shape.

You need to be able to:

- Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles
- Prove that two shapes are similar
- Understand the effect of enlargement on angles, perimeter, area and volume of shapes and solids
- Identify the scale factor of an enlargement of a similar shape
- Write the lengths, areas and volumes of two shapes as ratios in their simplest form
- Find missing lengths, areas and volumes in similar 3D solids
- Solve problems involving frustums of cones

Knowledge Organiser: 12 Similarity and Congruence in 2D and 3D

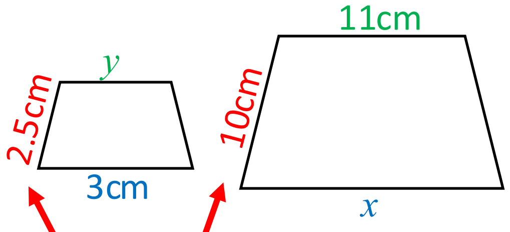
What you need to know:

Similarity - lengths

When similar shapes are an enlargement of one another a linear scale factor (lsf) is used. You can find the scale factor and then use this to find missing lengths on a shape.

When finding a missing length on the **larger** shape, **multiply** by the lsf.
When finding a missing length on the **smaller** shape, **divide** by the lsf.

Example – The trapezia are similar. Find the length of x and y



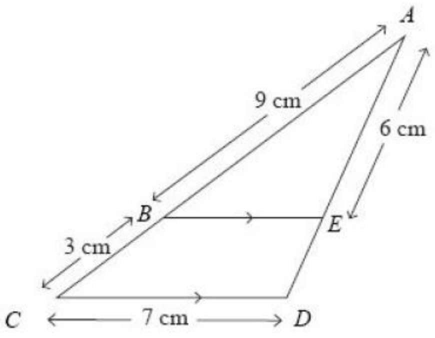
Choose two corresponding lengths to work out the scale factor

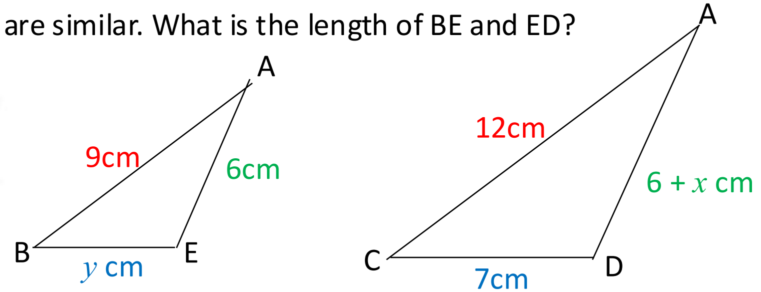
$$Scale\ factor = \frac{10}{2.5}$$
$$= 4$$

$$x = 3 \times 4$$
$$= \mathbf{12cm}$$

$$y = 11 \div 4$$
$$= \mathbf{2.75cm}$$

Example – Triangle ABE and ACD are similar. What is the length of BE and ED?





Split the diagram

$$Scale\ factor = \frac{12}{9}$$
$$= \frac{4}{3}$$

$$y = 7 \div \frac{4}{3}$$
$$= \mathbf{5.25cm}$$

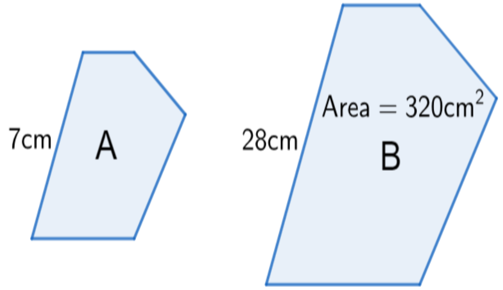
$$x + 6 = 6 \times \frac{4}{3}$$
$$x + 6 = 8$$
$$x = 8 - 6$$
$$\boldsymbol{x = 2cm}$$

Similarity – lengths, area and volume

If you know the scale factor for either the length, area or volume, you can work out the scale factors for all three.

Length = lsf
Area = asf = lsf^2
Volume = vsf = vsf^3

Example – Shape A and B are similar shapes. Work out the area of shape A



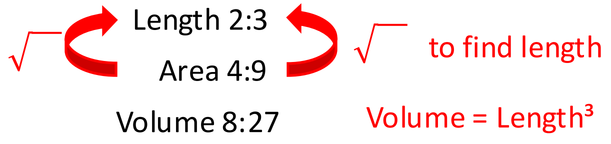
$$Scale\ factor = \frac{28}{7}$$
$$= 4$$

lsf= 4
asf = 4^2 = 16

Area of A = 320 ÷ 16
= **20cm^2**

Example – Solid C and solid D are mathematically similar. The ratio of the surface area of Solid C to the surface area of solid D is 4:9
The volume of solid D is 405cm^3.
Show that the volume of solid C is 120cm^3

Given area – find the ratios of the lengths and volume



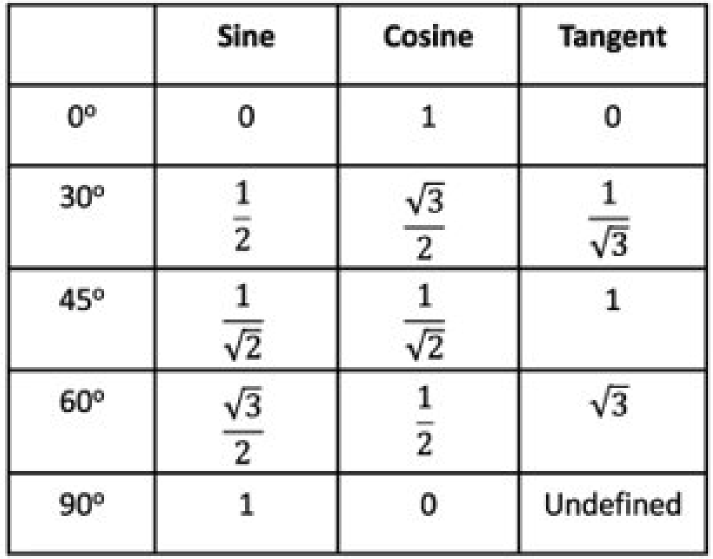
Use the ratios to find the volume

C = 8 x 15
Volume of C = **120cm^3**

Knowledge Organiser: 13a
Trigonometric Graphs and Exact Values

Key Concepts

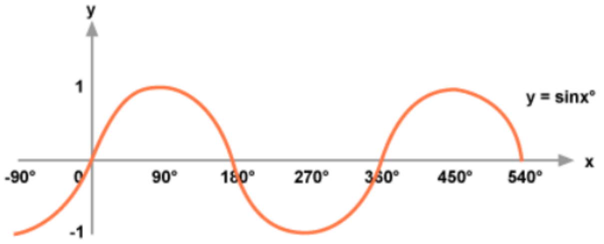
For some angles in a right-angled triangle, there is an exact trigonometric value. These are shown in the table below.

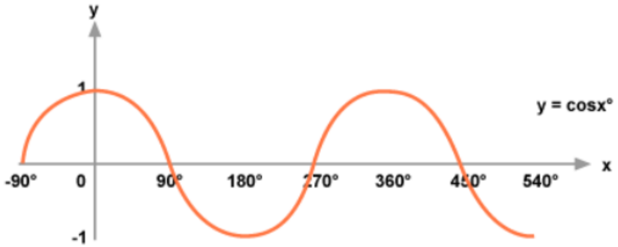


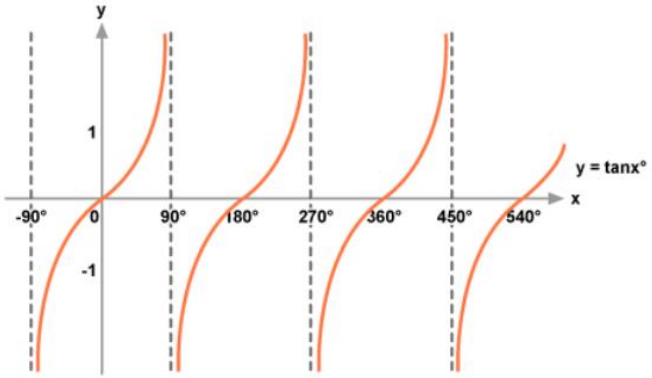
	Sine	Cosine	Tangent
0º	0	1	0
30º	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45º	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60º	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90º	1	0	Undefined

Examples

Trigonometric graphs

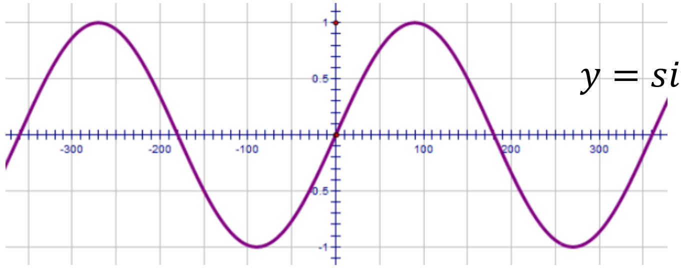






Key Words

Sine
Cosine
Tangent
Function
Angle
Theta θ



$y = sinx$

$sin30 = 0.5$

What other angles have a value of 0.5?

ANSWERS: -330º ,-210º , 150º

Knowledge Organiser: 13a
Transformations of Trigonometric Graphs

Key Concepts

All graphs can be transformed by applying different rules to their original function $y = f(x)$

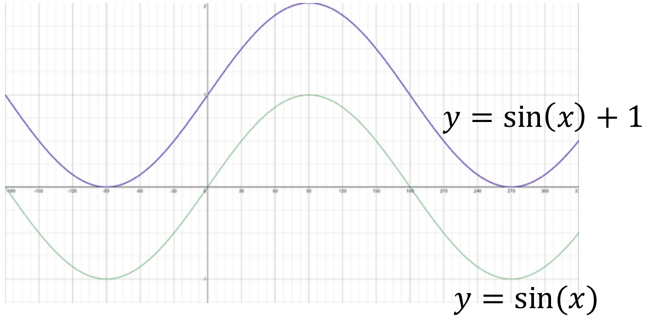
$\boldsymbol{y = -f(x)}$ This will reflect a function in the x axis.

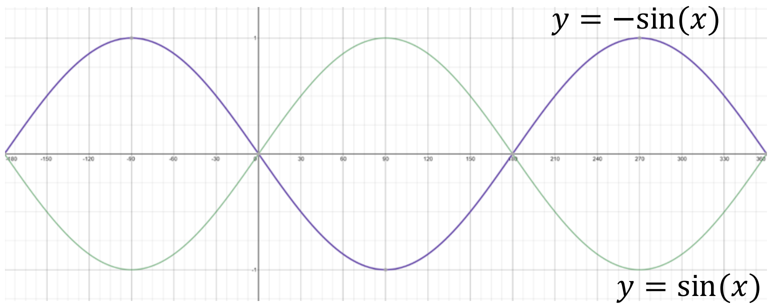
$\boldsymbol{y = f(-x)}$ This will reflect a function in the y axis.

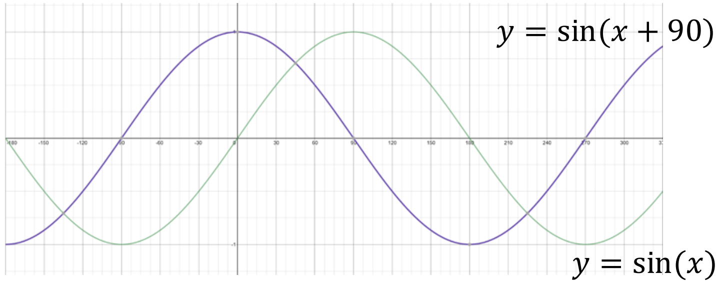
$\boldsymbol{y = f(x) \pm a}$ This will translate a function parallel to the y axis by $\begin{pmatrix} 0 \\ \pm a \end{pmatrix}$.

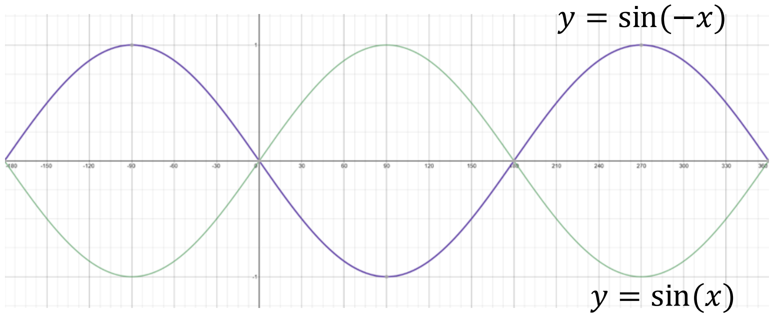
$\boldsymbol{y = f(x \pm a)}$ This will translate a function parallel to the x axis by $\begin{pmatrix} \mp a \\ 0 \end{pmatrix}$.

Examples









Key Words

Sine
Cosine
Tangent
Function
Transform
Translate
Reflect

1) Transform the graph of $y = \cos(x)$ by:
a) $y = \cos(x) - 1$ b) $y = -\cos(x)$

2) Transform the graph of $y = \tan(x)$ by:
a) $y = \tan(x + 90)$ b) $y = \tan(-x)$

ANSWERS 1a) Translate $\cos(x)$ down by 1 b) Reflect in the x axis 2a) Translate $\tan(x)$ left by 90
b) Reflect in the y axis

Knowledge Organiser: 13b
The Sine and Cosine Rules

Key Concepts

Sine rule

To calculate a missing side:

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

To calculate a missing angle:

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

Cosine rule

To calculate a missing side:

$$a^2 = b^2 + c^2 - 2bccosA$$

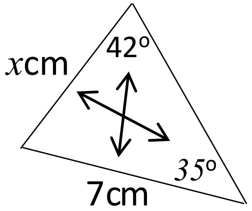
To calculate a missing angle:

$$cosA = \frac{b^2 + c^2 - a^2}{2bc}$$

Area of a triangle using sine

$$area = \frac{1}{2}absinC$$

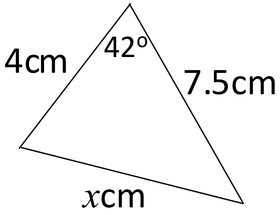
Examples



$$\frac{x}{sin35} = \frac{7}{sin42}$$

$$x = \frac{sin35 \times 7}{sin42}$$

$$x = 6.0cm$$

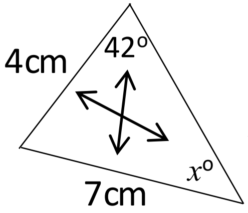


$$a^2 = b^2 + c^2 - 2bccosA$$

$$x^2 = 4^2 + 7.5^2 - 2 \times 4 \times 7.5 \times cos42$$

$$x^2 = 27.66$$

$$x = \sqrt{27.66} = 5.26cm$$

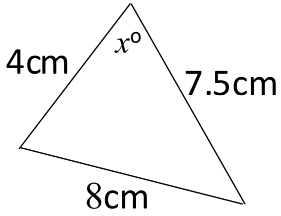


$$\frac{sinx}{4} = \frac{sin42}{7}$$

$$sinx = \frac{sin42 \times 4}{7}$$

$$x = sin^{-1}\left(\frac{sin42 \times 4}{7}\right)$$

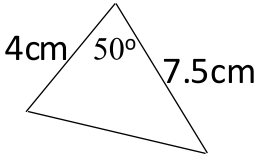
$$x = 22.5^o$$



$$cosA = \frac{4^2 + 7.5^2 - 8^2}{2 \times 4 \times 7.5}$$

$$A = cos^{-1}\left(\frac{4^2 + 7.5^2 - 8^2}{2 \times 4 \times 7.5}\right)$$

$$A = 82.1^o$$

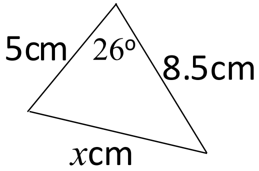


$$area = \frac{1}{2} \times 4 \times 7.5 \times sin50$$

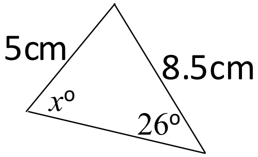
$$area = 11.49cm^2$$

Key Words

Sine
Cosine
Side
Angle
Inverse
2D



1a) Calculate x
b) Calculate the area of the triangle

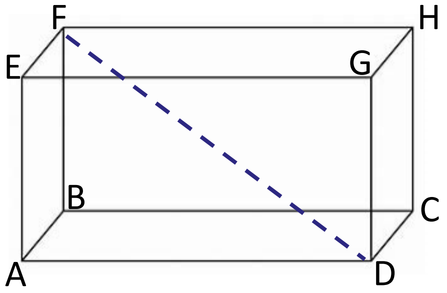


2a) Calculate x
b) Calculate the area of the triangle

ANSWERS 1a) 4.57cm b) 9.32cm² 2a) 48.18° b) 20.45cm²

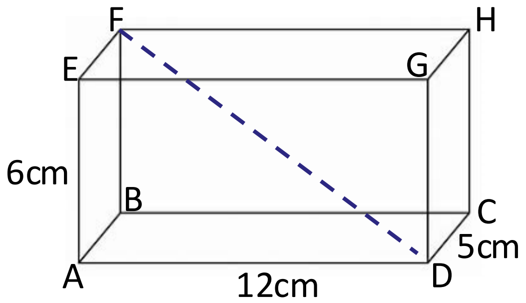
Knowledge Organiser: 13b
3D Trigonometry

Key Concepts

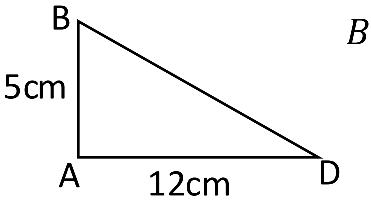


The **plane** of a cuboid is a flat 2-dimensional surface.
An example of a plane is ABCD.
An example of a **diagonal** in a cuboid is FD.

Examples

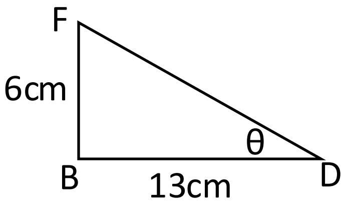


Calculate the length BD:



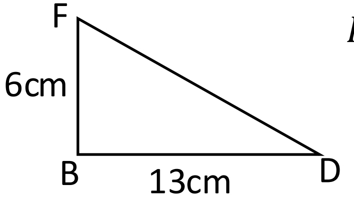
$$BD^2 = 12^2 + 5^2$$
$$BD = \sqrt{169}$$
$$BD = 13cm$$

Calculate the angle between FD and the plane ABCD:



$$tan\theta = \frac{6}{13}$$
$$\theta = tan^{-1}\left(\frac{6}{13}\right)$$
$$\theta = 24.78^o$$

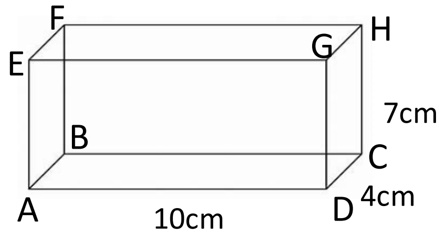
Calculate the length FD:



$$FD^2 = 13^2 + 6^2$$
$$FD = \sqrt{205}$$
$$FD = 14.32cm$$

Key Words

Sine
Cosine
Tangent
3D
Plane
Diagonal



1) Calculate the length AC
2) Calculate the length AH
3) Calculate the angle between AH and the plane ABCD.

ANSWERS 1) 10.77cm 2) 12.84cm 3) 33.02°

Knowledge Organiser: 14a Collecting Data

What you need to know

Stratified sampling

$$\text{Stratified sample size} = \frac{\text{Sample size}}{\text{Total population}} \text{ x Stratum size}$$

Example

There are 180 employees in a school. The table below shows the number of each type of employee.

Teachers	Teaching Assistants	Admin	Other
94	16	41	29

A stratified sample of size 50 is required

Calculate the number of each type of employee that should be chosen for the sample.

Teachers

$$\frac{94}{180} \text{ x } 50 = 26.1$$

Teaching Assistants

$$\frac{16}{180} \text{ x } 50 = 4.4$$

Admin

$$\frac{41}{180} \text{ x } 50 = 11.388$$

Other

$$\frac{29}{180} \text{ x } 50 = 8.055$$

Tip:

You must round these numbers to the nearest integer but check your total is still 50

Teachers	26
Teaching Assistants	5
Admin	11
Other	8

Key Terms:

Discrete data: data that can be categorised into a classification, there are a finite number of classifications e.g. hair colour.
Continuous data: data that can take any value. Data that is measured e.g. height or weight.

Qualitative data: data that describes something e.g. gender, race, hair colour …

Quantitative data: data that is in numerical form e.g. percentages or time.

Population: The entire group that is being studied.

Sample: A small selection of the population

You need to be able to:

- Understand primary and secondary data sources
- Understand what is meant by a sample and a population
- Understand how different sample sizes may affect the reliability of conclusions drawn
- Identify possible sources of bias and plan to minimise it
- Use stratified sampling to calculate sample sizes
- Use capture-recapture to estimate the size of a population

Knowledge Organiser: 14a Collecting Data

What you need to know:

Capture- Recapture

Capture- Recapture is a technique that can be used to estimate the total population when it is impractical to count the total population. A portion of the total population is captured, marked in some way and released. Later, another portion is captured and the number of marked individuals within that sample is counted.

Capture- Recapture formula

$$\frac{M}{N}=\frac{R}{T}$$

M = total marked
N = total population
R = number 'recaptured'
T = total captured on the 2nd visit

Tips:

- Input all the data you have into the equation first and then start to solve it.
- Show all steps of your working out.

Example

A group of students want to estimate the number of fish that live in a pond.

They catch 45 fish and tag them. They return all the fish to the pond.

They have captured the 45 fish (M), tagged them then released them.

The next day, they catch 60 fish. Of these, 18 are tagged.

They have now captured 60 fish in their second sample (T) and counted how many of them have been tagged the day before (R)

Estimate how many fish live in the pond.

We want to calculate N, we start by inputting all the data we have then solve the equation as we usually would.

$$\frac{45}{N}=\frac{18}{60}$$

$$\frac{45}{N}=0.3$$

$$\mathrm{N}=\frac{45}{0.3}=150$$

Assumptions made:

- No tags fall off
- No fish hatch/die (population remains constant)
- All fish have an equal chance of being caught

Knowledge Organiser: 14b Cumulative Frequency, Box Plots and Histograms

What you need to know:

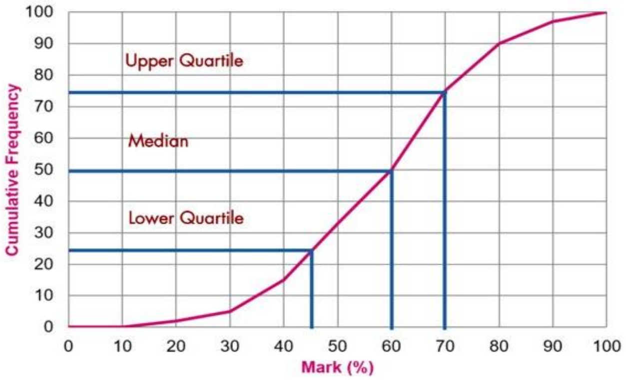
Cumulative Frequency Diagrams

Mark	Frequency	C.Frequency
$0 < x \leq 20$	4	4
$20 < x \leq 40$	11	15
$40 < x \leq 60$	35	50
$60 < x \leq 80$	40	90
$80 < x \leq 100$	10	100

The cumulative frequency is a running total of all the frequencies added together

Plot a cumulative frequency graph then find the median and the interquartile range.

To plot the graph, plot the end points of the inequality



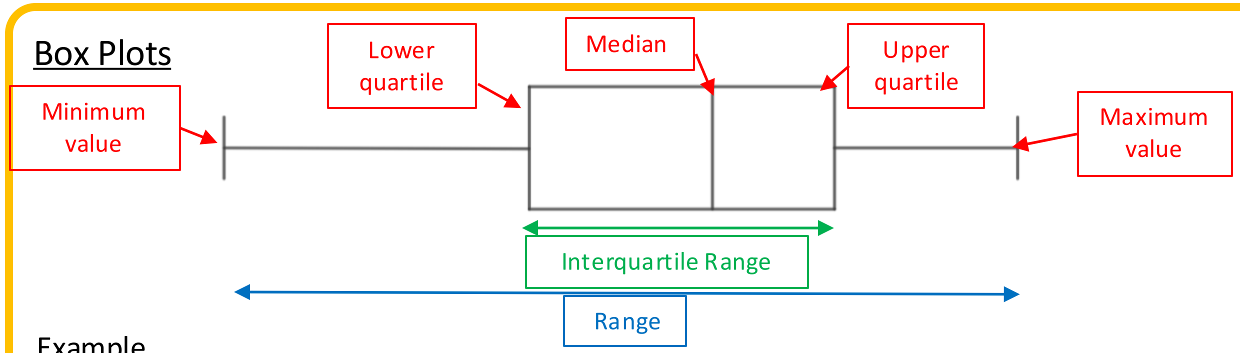
Lower Quartile = 25% of the way through the data
Median = 50% of the way through the data
Upper Quartile = 75% of the way through the data
Interquartile range = UQ – LQ

Lower Quartile = 45
Median = 60
Upper Quartile = 70
Interquartile range = 70 – 45 = 25

Key Terms:

Cumulative frequency: A running total of the frequencies, added up as you go along.
Box plot: A graphical way to display the median, quartiles, and extremes of a data set on a number line to show the distribution of the data.
Range: The largest number take away the smallest value in a set of data
Interquartile range: The difference between the upper and lower quartile.
Median: The middle value when a list of numbers is put in order from smallest to largest. A type of average.
Quartiles: The values that divide a list of numbers into quarters

Box Plots



Lower quartile

Median

Upper quartile

Minimum value

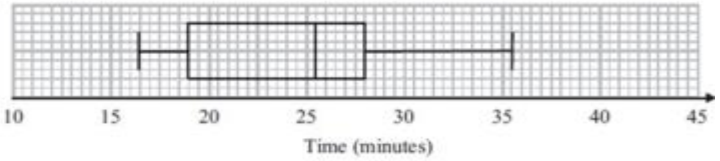
Maximum value

Interquartile Range

Range

Example

Read from the box plot the median, range and interquartile range.



Median = 26
Maximum value = 36, Minimum value = 17
Range = 36 – 17 = 19
LQ = 19, UQ = 28
IQR = 28 – 19 = 9

You need to be able to:

- Produce a cumulative frequency diagram.
- Read the median and the interquartile range from this graph.
- Draw and interpret a box plot.
- Calculate frequency densities and produce a histogram of both equal and unequal class widths.
- Read information from a histogram.

Knowledge Organiser: 14b Cumulative Frequency, Box Plots and Histograms

What you need to know:

Histograms

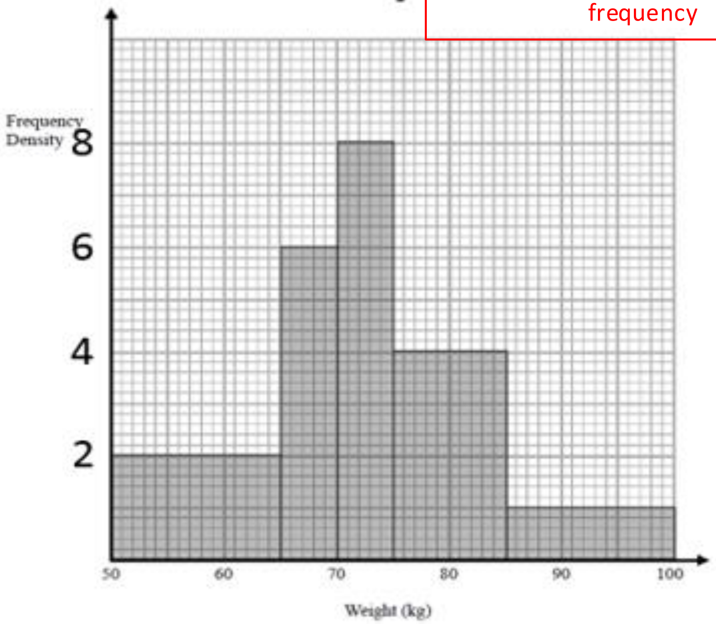
Example – Constructing a histogram

A group of people are weighed, and their results are recorded. Below is their data, draw a histogram to represent this data

Calculate the frequency densities then plot these as your Y axis

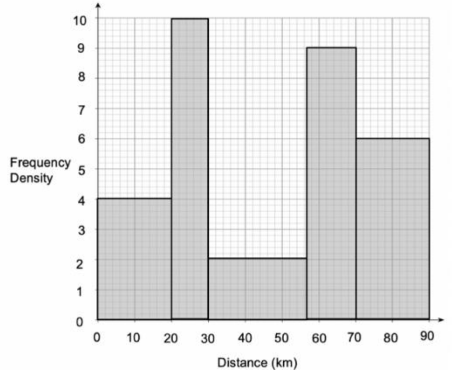
Weight	Frequency	Frequency density
50 < w ≤ 65	30	30 ÷ 15 = 2
65 < w ≤ 70	30	30 ÷ 5 = 6
70 < w ≤ 75	40	40 ÷ 5 = 8
75 < w ≤ 85	40	40 ÷ 10 = 4
85 < w ≤ 100	15	15 ÷ 15 = 1

The area of each bar on the histogram represents the total frequency

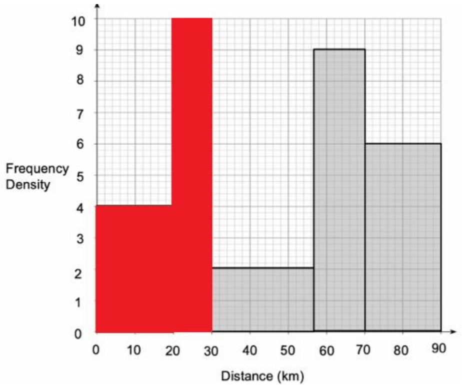


Example – Interpreting a histogram

Some cyclists from a local cycling club go out for their usual Sunday ride. There are many different lengths of routes to suit cyclists of all abilities. The histogram below shows this information.



Estimate the number of cyclists who rode for 30 kilometres or less.



Frequency = Frequency density x class width

Frequency for 0 – 20 = 4 x 20 = 80
Frequency for 20 – 30 = 10 x 10 = 100

Number of cyclists who rode for 30km or less is 100 + 80 = 180

Key Terms:

Histogram: Graphical display of continuous data using bars of different heights, where the area of each bar represents the frequency

Class width: The range of each group of data.
E.g. 10≤ x < 22 has a class width = 12

Frequency density: is defined as the ratio of the frequency of a class to its width.

$$\text{Frequency density} = \frac{Frequency}{Class\ width}$$

Frequency = Frequency density x class width

Knowledge Organiser: 15
Expanding and Factorising

Key Concepts

Expanding brackets
Where every term inside each bracket is multiplied by every term all other brackets.

Factorising expressions
Putting an expression back into brackets. To "factorise fully" means take out the HCF and there are often 2 common terms.

Difference of two squares
When two brackets are repeated with the exception of a sign change. All numbers in the original expression will be square numbers.

Examples

Expand and simplify:

1) $4(m + 5) + 3$

$= 4m + 20 + 3$

$= 4m + 23$

2) $(p + 2)(2p - 1)$

$= p^2 + 4p - p - 2$

$= p^2 + 3p - 2$

3) $(p + 3)(p - 1)(p + 4)$

$= (p^2 + 3p - p - 3)(p + 4)$

$= (p^2 + 2p - 3)(p + 4)$

$= p^3 + 4p^2 + 2p^2 + 8p - 3p - 12$

$= p^3 + 6p^2 + 5p - 12$

Factorise fully:

1) $16at^2 + 12at \quad = 4at(4t + 3)$

2) $x^2 - 2x - 3 \quad = (x - 3)(x + 1)$

3) $6x^2 + 13x + 5$

$= \underbrace{6x^2 + 3x} + \underbrace{10x + 5}$

$= 3x(2x + 1) + 5(2x + 1)$

$= (3x + 5)(2x + 1)$

4) $4x^2 - 25$

$= (2x + 5)(2x - 5)$

Key Words

Expand
Factorise fully
Bracket
Difference of two squares

A)Expand:

1) $5(m - 2) + 6$ 2) $(5g - 4)(2g + 1)$ 3) $(y + 1)(y - 2)(y + 3)$

B) Factorise:

1) $5b^2c - 10bc$ 2) $x^2 - 8x + 15$ 3) $3x^2 + 8x + 4$ 4) $9x^2 - 25$

ANSWERS: A 1) $5m - 4$ 2) $10g^2 - 3g - 4$ 3) $y^3 + 2y^2 - 5y - 6$
B 1) $5bc(b - 2)$ 2) $(x - 3)(x - 5)$ 3) $(3x + 2)(x + 2)$ 4) $(3x + 5)(3x - 5)$

Knowledge Organiser: 15
Solving Quadratics

Key Concepts

We can solve quadratic equations in 4 different ways:

$$ax^2 + bx + c = 0$$

Factorising – put into brackets first

Completing the square

$$\left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2 = 0$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graphically

Examples

Factorising:

$$x^2 + 7x + 10 = 0$$

$$(x + 2)(x + 5) = 0$$

$$Either: x + 2 = 0$$

$$x = -2$$

$$Or: \ x + 5 = 0$$

$$x = -5$$

Completing the square – leave your answer in root form:

$$x^2 + 6x + 5 = 0$$

$$\left(x + \frac{6}{2}\right)^2 + 5 - \left(\frac{6}{2}\right)^2 = 0$$

$$(x + 3)^2 + 5 - 3^2 = 0$$

$$(x + 3)^2 - 4 = 0$$

$Either: x = \sqrt{4} - 3$ x=-1

$Or: \ x = -\sqrt{4} - 3$ x = -5

Quadratic formula – give your answer to 2 decimal places:

$$x^2 + 4x - 2 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 + 8}}{2}$$

$$Either: \ x = 0.45$$

$$Or: \ x = -4.45$$

Key Words

Solve

Quadratic

Equation

Factorise

Completing the Square

Quadratic formula

1) Solve by factorising: $x^2 + 6x + 8 = 0$
2) Solve by completing the square: $x^2 + 8x + 10 = 0$
3) Solve by using the quadratic formula: $x^2 + 9x - 1 = 0$

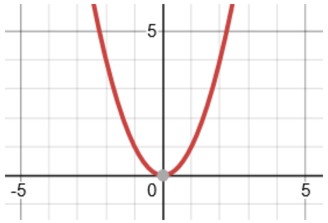
ANSWERS: 1) $x = -2, x = -4$ 2) $x = \sqrt{6} - 4, x = -\sqrt{6} - 4$ 3) $x = 0.11, x = -9.11$

Knowledge Organiser: 15
Quadratic Graphs

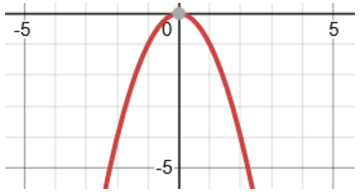
Key Concepts

A quadratic graph will always be in the shape of a parabola.

$y = x^2$

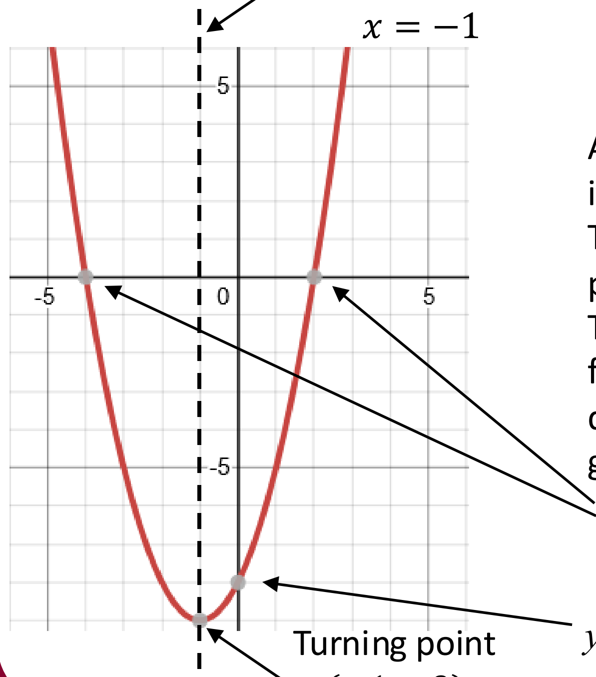


$y = -x^2$



The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.

Examples



Line of symmetry
$x = -1$

$y = x^2 + 2x - 8$

A quadratic equation can be solved from its graph.
The roots of the graph tell us the possible solutions for the equation.
There can be 1 root, 2 roots or no roots for a quadratic equation. This is dependent on how many times the graph crosses the x axis.

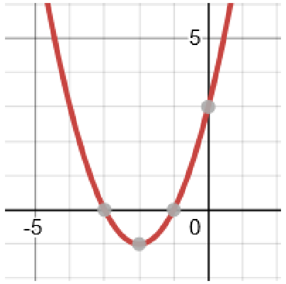
Roots $x = -4$
$x = 2$

y intercept = -8

Turning point
$(-1, -9)$

Key Words

Quadratic
Roots
Intercept
Turning point
Line of symmetry



Identify from the graph of $y = x^2 + 4x + 3$:
1) The line of symmetry
2) The turning point
3) The y intercept
4) The two roots of the equation

ANSWERS 1) $x = -2$ 2) $(-2, -1)$ 3) 3 4) $x = -1$ and $x = -3$

Knowledge Organiser: 15
Solving Simultaneous Equations Graphically

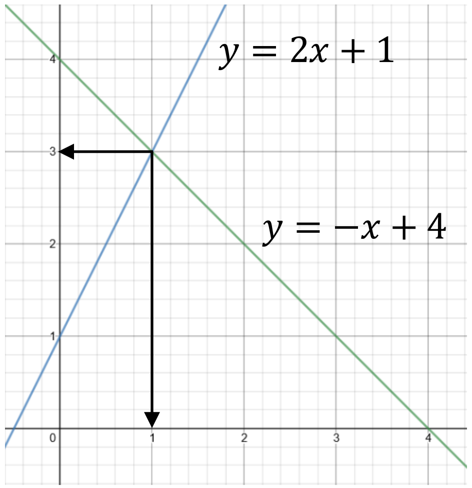
Key Concepts

Simultaneous equations are when **more than one equation** are given which involve **more than one variable**. The variables have the **same value** in each equation.

Simultaneous equations can be solved **graphically** whereby the **intersection** of the graphs gives the x and y values.

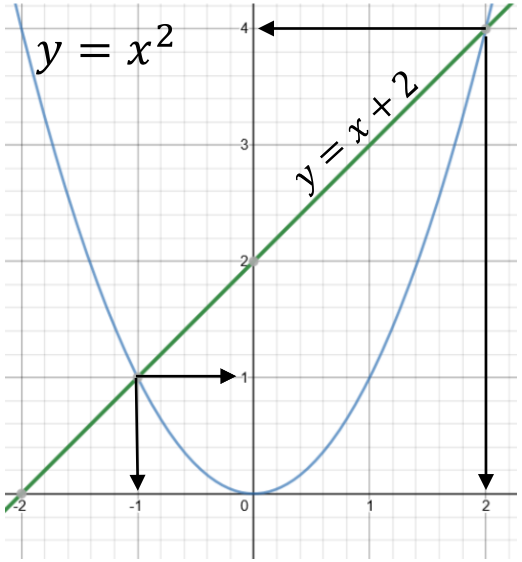
Examples

Solve graphically: $y = 2x + 1$
$y = -x + 4$



$x = 1$ and $y = 3$

Solve graphically: $y = x^2$
$y = x + 2$



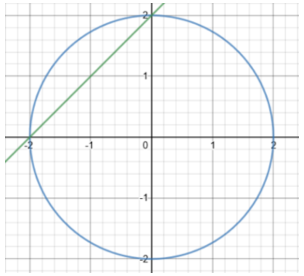
$x = -1$ and $y = 1$

$x = 2$ and $y = 4$

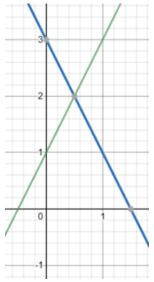
Key Words

Simultaneous
Equation
Intersection

1)



2)



Solve each set of simultaneous equations graphically.

ANSWERS 1) $x = -2$ and $y = 0$, $x = 0$ and $y = 2$ 2) $x = 0.5$ and $y = 2$

Knowledge Organiser: 15
Solving Quadratic Inequalities Graphically

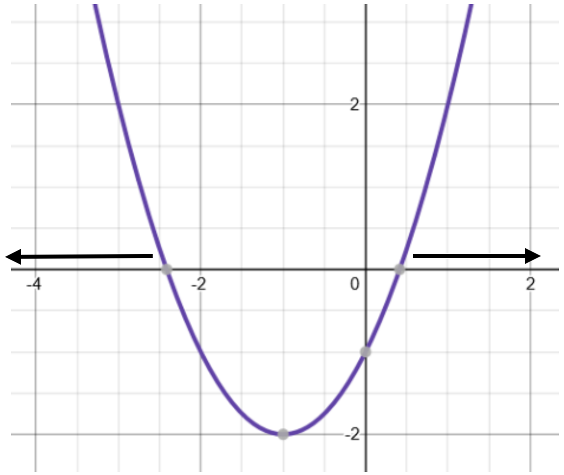
Key Concepts

When a quadratic inequality is solved it provides the **range of values** that are possible.

When an equation has **solutions greater than 0** then the solutions are taken from **above the x axis**.

When an equation has solutions **less than 0** then the solutions are taken from **below the x axis**.

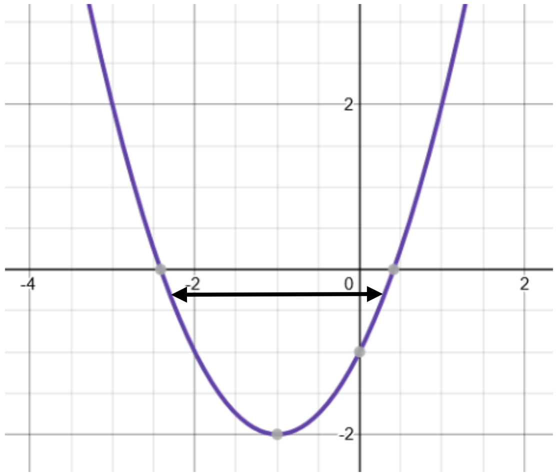
Examples



$x^2 + 2x - 1 \geq 0$

The range of solutions are:

$x \geq 0.5 \; and \; x \leq -2.5$



$x^2 + 2x - 1 \leq 0$

The range of solutions are:

$-2.5 \leq x \leq 0.5$

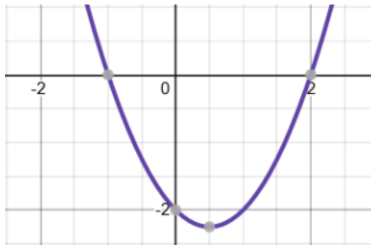
Key Words

Quadratic

Inequality

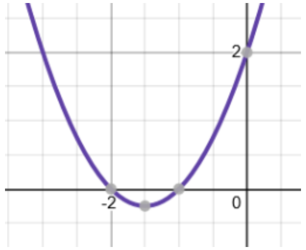
Solutions

1)



$x^2 - x - 2 \geq 0$

2)



$x^2 + 3x + 2 \leq 0$

State the range of values possible for each inequality.

ANSWERS 1) $x \leq -1 \; and \; x \geq 2$ 2) $-2 \leq x \leq -1$

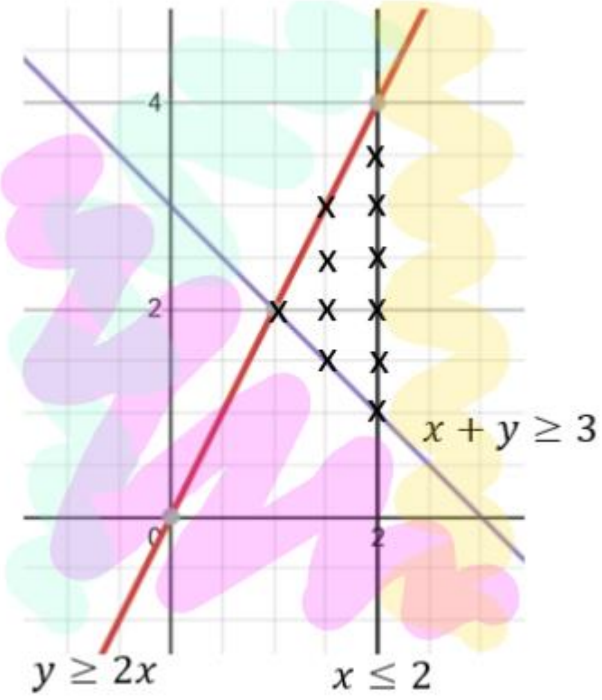
Knowledge Organiser: 15
Inequalities and Regions on Graphs

Key Concepts

Inequalities can be represented on graphs. They highlight regions where all of the possible true values exist within its given constraints.

Solid lines are used on the graph when ≤ ≥ are involved.
Dashed lines are used on the graph when <> are involved.

Examples

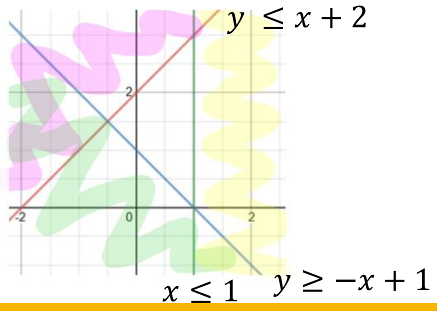


Highlight the region that represents the possible solutions to the following inequalities:

$$x \leq 2$$
$$y \leq 2x$$
$$x + y \geq 3$$

- Plot the points for each linear graph and draw the lines.
- Choose a coordinate whose values can be substituted in to each inequality. If the inequality is FALSE once values have been substituted in, then shade this area.
- You should be left with one area with no shading once you have repeated this for all inequalities.
- Each x is a possible set of integer values which are true for all three inequalities. Eg. (2,1), (2,2) (2,3)…

Key Words
Region
Inequality
Solution



State the possible integer values that are true for the given inequalities.

ANSWER (1,0), (1,1), (1,2), (1,3), (0,1), (0,2)

Knowledge Organiser: 15

Iteration

Key Concepts

Iteration is the **repetition** of a mathematical procedure applied to the result of a previous application, typically as a means of **obtaining successively closer approximations** to the solution of a problem.

Examples

When $x_0 = 0$ $\quad x_{n+1} = \dfrac{2}{x_n^2 + 3}$

Calculate the values of x_1, x_2, x_3 to find an estimate for the solution to $x^3 + 3x = 2$

$$x_{0+1} = \frac{2}{0^2 + 3} = 0.\dot{6}$$

$$x_{1+1} = \frac{2}{0.\dot{6}^2 + 3} = 0.5806451613$$

$$x_{2+1} = \frac{2}{(0.58\ldots)^2 + 3} = 0.5993140006$$

We substitute this value into the next step.

An estimate of the solution is 0.6 because all of the solutions round to 1d.p.

Key Words

Iteration

Solution

Approximate

Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \sqrt{\dfrac{3}{2x_n}}$ four times to find an estimate for the solution.

ANSWER 1.1 to 1d.p.

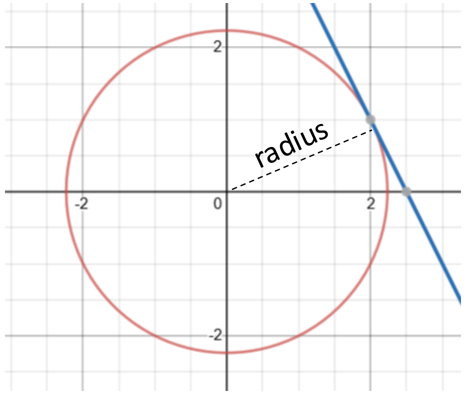
Knowledge Organiser: 15
Tangent to a Circle

Key Concepts

A **tangent** touches a circle at **one point**.

A **tangent** line is **perpendicular** to the **radius** of the circle.

The gradient of the tangent is the **negative reciprocal** of the gradient of the equation of the line of the radius.



Find the equation of the tangent to the circle with equation:

$$x^2 + y^2 = 5$$

which passes through the point (2,1).

Examples

1) Find the equation of the line which is the radius of the circle.

$$gradient = \frac{1}{2} \quad therefore\ y = \frac{1}{2}x$$

2) The tangent is perpendicular to the radius.

$$gradient\ of\ tangent = negative\ reciprocal\ of\ \frac{1}{2}$$
$$= -2$$

3) Substitute in the given coordinate (2,1) to $y = -2x + c$

$$y = -2x + c$$
$$1 = (-2 \times 2) + c$$
$$1 + 4 = c$$
$$5 = c$$
$$y = -2x + 5$$

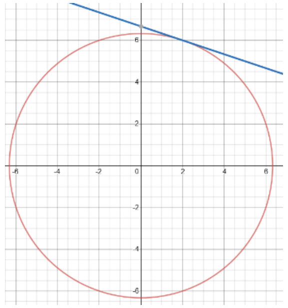
Key Words

Radius
Tangent
Negative reciprocal
Perpendicular
Gradient

Find the equation of the tangent to the circle with equation:

$$x^2 + y^2 = 40$$

which passes through the point (2,6).



ANSWER $y = -\frac{1}{3}x + \frac{20}{3}$

Knowledge Organiser: 15
Equation of a Circle

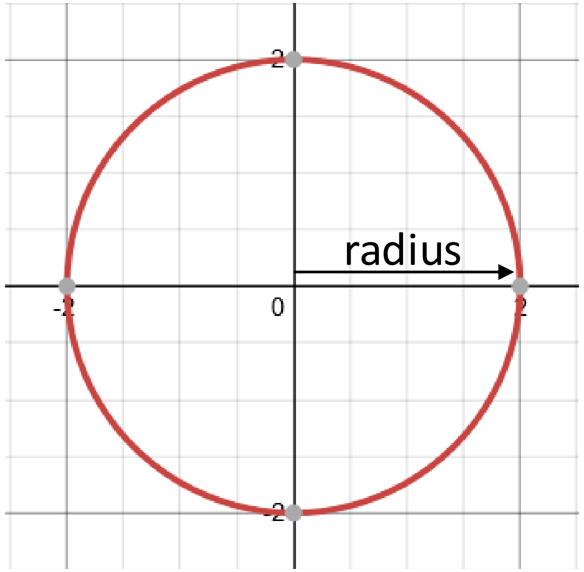
Key Concepts

The **equation of a circle** will be in the format:

$$x^2 + y^2 = radius^2$$

The **centre** of each circle will be at the coordinate **(0,0).**

Examples



$$x^2 + y^2 = 4$$

$$Radius = \sqrt{4}$$
$$= \pm 2$$

Therefore we can plot the following coordinates to support us sketching our graph: (0,2), (0,-2), (2,0), (-2,0)

Key Words

Radius
Centre
Sketch
Square root

Calculate the length of the radius for each of the following equations of circles:

1) $x^2 + y^2 = 25$

2) $x^2 + y^2 = 49$

3) $x^2 + y^2 = 256$

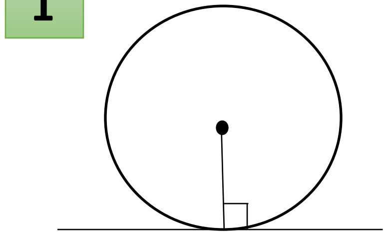
4) $x^2 + y^2 = 22$

ANSWER 1) 5 2) 7 3) 16 4) $\sqrt{22}$

Knowledge Organiser: 16a Circle Theorems

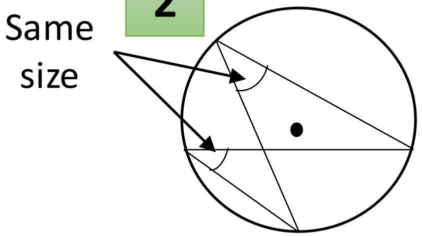
What you need to know: Specific angle rules inside circles and correct language for reasons

1



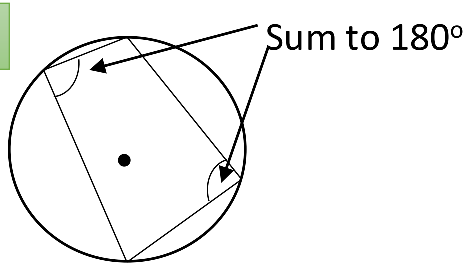
The angle between a radius and a tangent is 90°

2



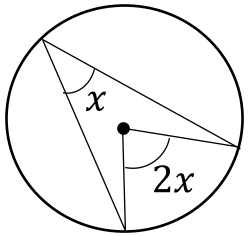
Angles in the same segment are equal

3



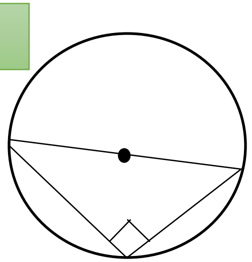
Opposite angles in a cyclic quadrilateral add to 180°

4



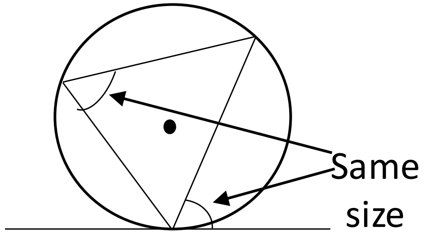
The angle at the centre is twice that at the circumference

5



The angle at the circumference in a semi-circle is 90°

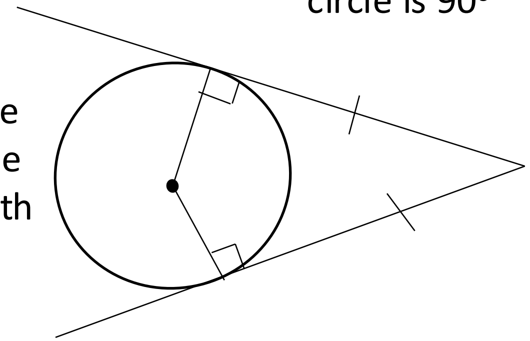
6



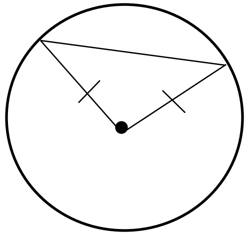
Alternate segment theorem

7

Two tangents from the same point outside the circle are equal in length (kite shape)



8



Two radii and a chord inside a circle create an isosceles triangle

Key Terms:

Radius/ Radii(pl.)
Centre
Tangent
Circumference
Right angle
Segment
Alternate

Cyclic-all vertices touch the circumference of the circle

Theory/theorem – a rule that applies in all cases

Key Facts:

Try look, cover, write, check to be able to identify and describe each of the 8 circle theorems.

1. Read through the theorems to memorise the key vocabulary and distinguishing shapes
2. Cover them over
3. Attempt to recreate them on another sheet of paper
4. Check how many you remembered perfectly.
5. Try again until you have all 8.

Knowledge Organiser: 17
Algebraic Fractions

Topic/Skill	**Definition/Tips**	**Topic: Algebraic Fractions** **Example**
1. Algebraic Fraction	A fraction whose **numerator** and **denominator** are **algebraic expressions.**	$\frac{6x}{3x-1}$
2. Adding/ Subtracting Algebraic Fractions	For $\frac{a}{b} \pm \frac{c}{d}$, the **common denominator** is bd $\frac{a}{b} \pm \frac{c}{d} = \frac{ad}{bd} \pm \frac{bc}{bd} = \frac{ad \pm bc}{bd}$	$\frac{1}{x} + \frac{x}{2y}$ $= \frac{1(2y)}{2xy} + \frac{x(x)}{2xy}$ $= \frac{2y + x^2}{2xy}$
3. Multiplying Algebraic Fractions	**Multiply** the **numerators together** and the **denominators together**. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$	$\frac{x}{3} \times \frac{x+2}{x-2}$ $= \frac{x(x+2)}{3(x-2)}$ $= \frac{x^2+2x}{3x-6}$
4. Dividing Algebraic Fractions	**Multiply** the first fraction by the **reciprocal of the second fraction**. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$	$\frac{x}{3} \div \frac{2x}{7}$ $= \frac{x}{3} \times \frac{7}{2x}$ $= \frac{7x}{6x} = \frac{7}{6}$
5. Simplifying Algebraic Fractions	**Factorise** the numerator and denominator and **cancel common factors.**	$\frac{x^2+x-6}{2x-4} = \frac{(x+3)(x-2)}{2(x-2)} = \frac{x+3}{2}$

Knowledge Organiser: 17
Equation of a Circle

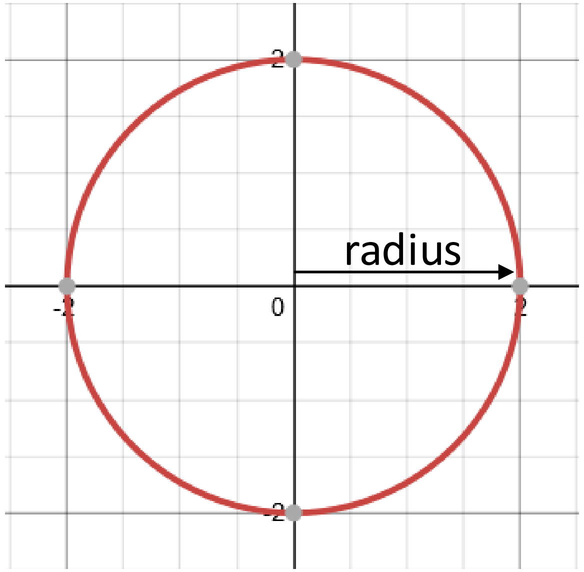
Key Concepts

The **equation of a circle** will be in the format:

$$x^2 + y^2 = radius^2$$

The **centre** of each circle will be at the coordinate **(0,0).**

Examples



$$x^2 + y^2 = 4$$

$$Radius = \sqrt{4}$$
$$= \pm 2$$

Therefore we can plot the following coordinates to support us sketching our graph: (0,2), (0,-2), (2,0), (-2,0)

Key Words

Radius
Centre
Sketch
Square root

Calculate the length of the radius for each of the following equations of circles:

1) $x^2 + y^2 = 25$
2) $x^2 + y^2 = 49$
3) $x^2 + y^2 = 256$
4) $x^2 + y^2 = 22$

ANSWER 1) 5 2) 7 3) 16 4) $\sqrt{22}$

Knowledge Organiser
Surds

Key Concepts

Surds are irrational numbers that cannot be simplified to an integer from a root.

Examples of a surd:

$\sqrt{3}, \sqrt{5}, 2\sqrt{6}$

Examples

Simplify:

$$\begin{aligned} 4\sqrt{20} \times 2\sqrt{3} &= 8\sqrt{20 \times 3} \\ &= 8\sqrt{60} \\ &= 8\sqrt{4}\sqrt{15} \\ &= 16\sqrt{15} \end{aligned}$$

$$\begin{aligned} 3\sqrt{40} \div \sqrt{2} &= 3\sqrt{40 \div 2} \\ &= 3\sqrt{20} \\ &= 3\sqrt{4}\sqrt{5} \\ &= 6\sqrt{5} \end{aligned}$$

Simplify:

$$\begin{aligned} \sqrt{3}(5 + \sqrt{6}) &= 5\sqrt{3} + \sqrt{18} \\ &= 5\sqrt{3} + \sqrt{9}\sqrt{2} \\ &= 5\sqrt{3} + 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} (3 + \sqrt{2})(4 + \sqrt{12}) &= 12 + 4\sqrt{2} + 3\sqrt{12} + \sqrt{24} \\ &= 12 + 4\sqrt{2} + 3\sqrt{4}\sqrt{3} + \sqrt{4}\sqrt{6} \\ &= 12 + 4\sqrt{2} + 6\sqrt{3} + 2\sqrt{6} \end{aligned}$$

Key Words

Rational

Irrational

Surd

Simplify fully:

1) $2\sqrt{27}$ 2) $2\sqrt{18} \times 3\sqrt{2}$ 3) $\sqrt{72}$ 4) $12\sqrt{56} \div 6\sqrt{8}$

5) $3\sqrt{2}(5 - 2\sqrt{8})$ 6) $(2 + \sqrt{5})(3 - \sqrt{5})$

ANSWERS: 1) $6\sqrt{3}$ 2) 36 3) $6\sqrt{2}$ 4) $2\sqrt{7}$ 5) $15\sqrt{2} - 24$ 6) $1 + \sqrt{5}$

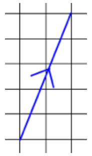
Knowledge Organiser: 18 Vectors and Geometric Proof

What you need to know:

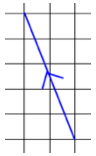
Column Vectors describe movements and forces

$\begin{pmatrix} x \\ y \end{pmatrix}$

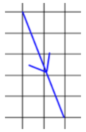
The top number is left or right (+ right, - left)
The bottom number is up or down (+ up, - down)



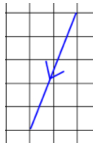
$\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ means 2 right 5 up



$\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ means 2 left 5 up



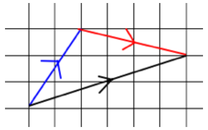
$\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ means 2 right 5 down



$\begin{pmatrix} -2 \\ -5 \end{pmatrix}$ means 2 left 5 down

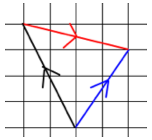
Add and Subtract Vectors and Multiply Vectors by Scalars

You can add and subtract vectors or multiply them by a scalar.



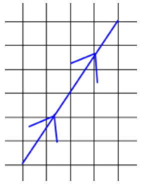
$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 2+4 \\ 3+-1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

To add vectors you add the x components and the y components.



$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 2-4 \\ 3--1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

To subtract vectors you subtract the x components and the y components.



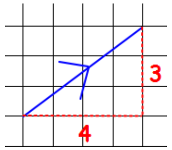
$$2 \times \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \times 2 \\ 2 \times 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

To multiply a vector by a scalar you multiply the x component and the y component.

Key Terms:

Vector: A quantity that has a magnitude (size) and direction.
Direction: The way a vector points.
Magnitude: The length (size) of a vector.
Scalar: A quantity that has a magnitude but no direction (e.g. a number, not a vector).
Parallel: Lines that never meet (have the same direction).
Collinear: Points that lie on a straight line.

The Magnitude of a Vector



The magnitude of a vector is the length (size) of the vector. To work out the magnitude of a vector you need to use Pythagoras' theorem.

To work out the magnitude of the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$:

$$4^2 + 3^2 = x^2$$
$$16 + 9 = x^2$$
$$25 = x^2$$
$$x = \sqrt{25} = 5$$

So the magnitude is 5

Parallel Vectors

Multiplying a vector by a scalar gives a parallel vector.

e.g. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is parallel to $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

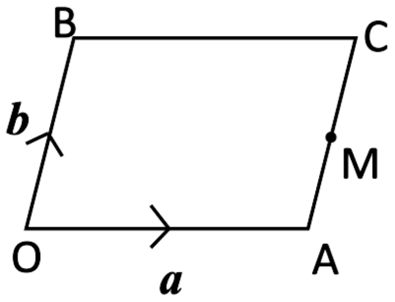
You need to:

- Understand column vector notation.
- Know that 2a is parallel to a and twice its length.
- Draw column vectors.
- Add and subtract column vectors.
- Multiply column vectors by a scalar.
- Find the magnitude (length) of a vector.
- Solve geometrical problems in 2D.
- Prove points are collinear and prove lines/vectors are parallel

Knowledge Organiser: 18 Vectors and Geometric Proof

What you need to know:

Simple Geometric Problems:



$OACB$ is a parallelogram, M is the midpoint of AC

$\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$

Write, in terms of a and b

- i) the vector $\overrightarrow{BC}$
- ii) the vector $\overrightarrow{BO}$
- iii) the vector $\overrightarrow{OC}$
- iv) the vector $\overrightarrow{OM}$

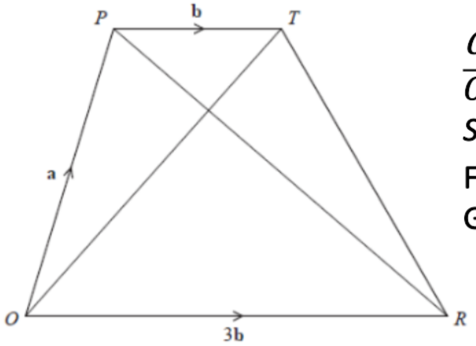
i) $\overrightarrow{BC} = a$ because BC is parallel to $\overrightarrow{OA}$ and is the same length

ii) $\overrightarrow{BO} = -b$ because $\overrightarrow{BO}$ is the same as $\overrightarrow{OB}$ in the opposite direction

iii) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = a + b$ (or $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = b + a$)

iv) $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = a + \frac{1}{2}b$ because M is the midpoint of AC and $\overrightarrow{AC}$ =b

Geometric Problems Involving Ratio:



$OPTR$ is a trapezium,

$\overrightarrow{OP} = a$, $\overrightarrow{PT} = b$ and $\overrightarrow{OR} = 3b$

S is the point on PR such that $PS : SR = 1 : 3$

Find $\overrightarrow{OS}$ in terms of a and b

Give your answer in its simplest form.

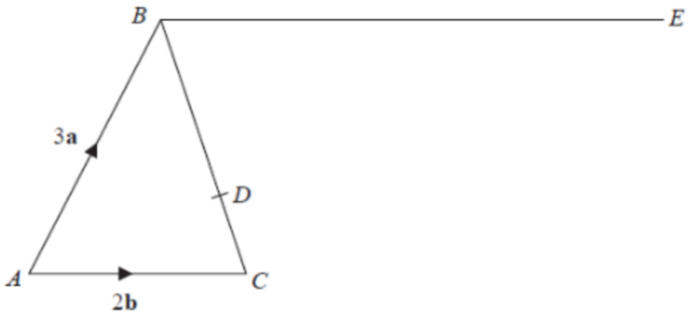
$\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{PS} = \overrightarrow{OP} + \frac{1}{4}\overrightarrow{PR}$ because $PS = \frac{1}{4}PR$ (using the ratio 1 : 3)

$\overrightarrow{OP} = a$

$\frac{1}{4}\overrightarrow{PR} = \frac{1}{4}(-a + 3b) = -\frac{1}{4}a + \frac{3}{4}b$ because $\overrightarrow{PR} = -a + 3b$

$\overrightarrow{OS} = a - \frac{1}{4}a + \frac{3}{4}b = \frac{3}{4}a + \frac{3}{4}b$

Parallel Lines and Collinear Points:



ABC is a triangle,

$\overrightarrow{AB} = 3a$, $\overrightarrow{AC} = 2b$ and $\overrightarrow{BE} = 3\overrightarrow{AC}$

D is the point on BC such that $BD : DC = 3 : 1$

Prove that ADE is a straight line.

This is the same as proving that the points A, D and E are collinear.

To do this you need to show that any two vectors made from the points A, D and E are parallel, i.e. that one vector is a multiple of the other.

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AB} + \frac{3}{4}\overrightarrow{BC}$ because BD= $\frac{3}{4}BC$ (using the ratio 3 : 1)

$\overrightarrow{AB} = 3a$

$\frac{3}{4}\overrightarrow{BC} = \frac{3}{4}(-3a + 2b) = -\frac{9}{4}a + \frac{3}{2}b$ because $\overrightarrow{BC} = -3a + 2b$

$\overrightarrow{AD} = 3a - \frac{9}{4}a + \frac{3}{2}b = \frac{3}{4}a + \frac{3}{2}b$

$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = \overrightarrow{AB} + 3\overrightarrow{AC} = 3a + 6b$

$\overrightarrow{AE} = 4\overrightarrow{AD}$ so ADE is a straight line (points A, D and E are collinear) because AE is parallel to AD and have point A in common

Knowledge Organiser: 19a
Straight Line Graphs and Equation of a Line

Key Concepts

Coordinates in 2D are written as follows:

x is the value that is to the left/right → (x, y) ← y is the value that is to up/down

Straight line graphs always have the equation:

$$y = mx + c$$

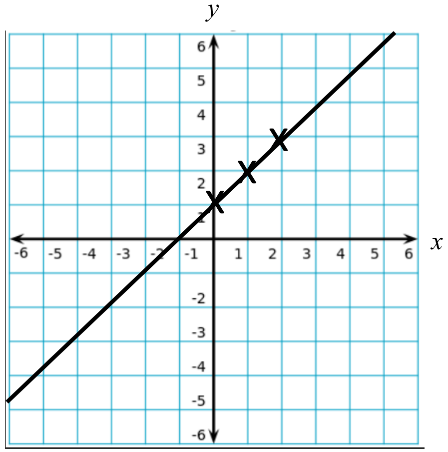
m is the **gradient** i.e. the steepness of the graph.

c is the **y intercept** i.e. where the graph cuts the y axis.

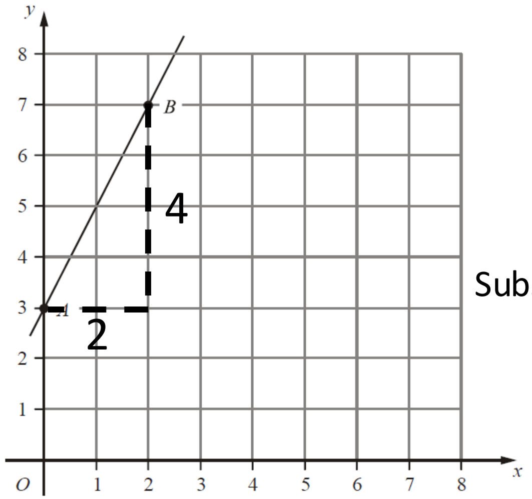
Examples

Plot the graph of $y = x + 1$

x	**0**	**1**	**2**
y	1	2	3



Calculate the equation of this line:



$$y = mx + c$$

$$m = \frac{4}{2}$$

$$= 2$$

$$y = 2x + c$$

Substitute in a coordinate: (2,7)

$$7 = (2 \times 2) + c$$

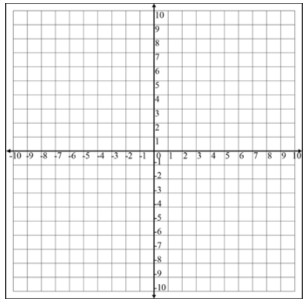
$$3 = c$$

$$y = 2x + 3$$

Key Words

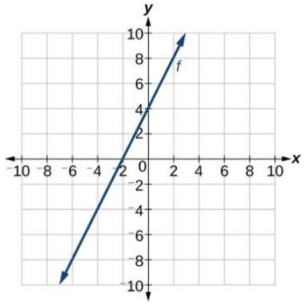
Coordinate

Gradient



1) Plot the line $y = 3x - 2$

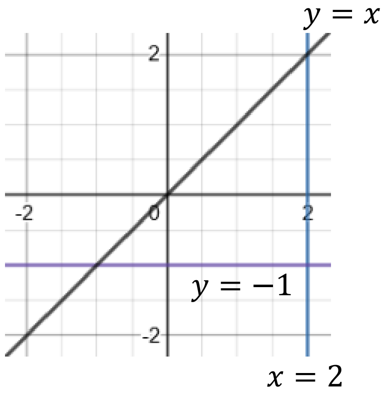
2) Find the equation of the line for the attached graph.



ANSWERS: 2) $y = 2x + 4$

Knowledge Organiser: 19a
Coordinate Geometry

Key Concepts



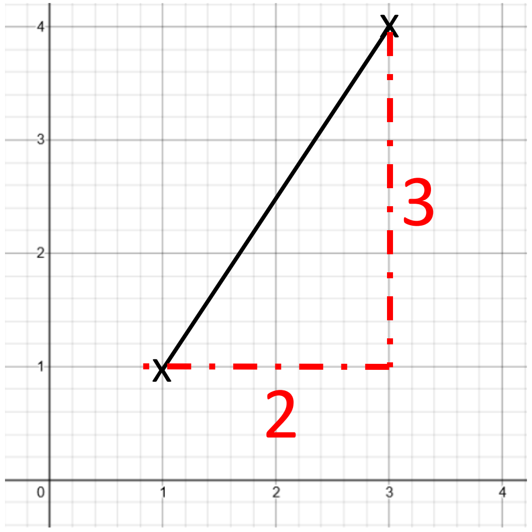
Parallel lines have the same gradient.

e.g. $y = 2x + 3 \text{ and } y = 2x - 1$

Perpendicular line gradients are the negative reciprocal of one another

e.g. $y = 2x \text{ and } y = -\frac{1}{2}x$

Examples



Calculate the midpoint between the coordinates (1,1) and (3,4).

$$Midpoint = \left(\frac{3+1}{2}, \frac{1+4}{2}\right) = (2, 2.5)$$

Calculate the distance between the coordinates (1,1) and (3,4).

$$Length = \sqrt{2^2 + 3^2}$$

$$= 3.61$$

Key Words

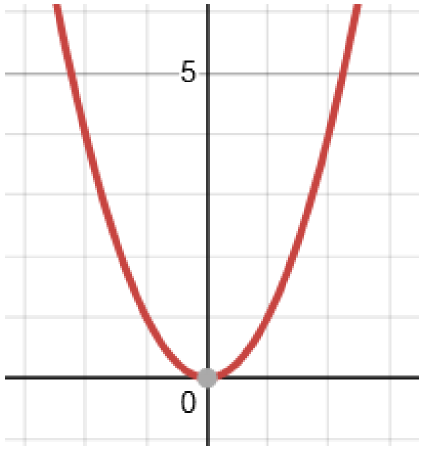
Parallel
Perpendicular
Gradient
Midpoint
Length

1) Calculate the midpoint between the coordinates (2,7) and (9,11).
2) Calculate the distance between the coordinates (2,7) and (9,11).

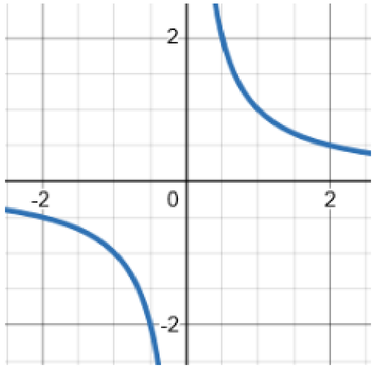
ANSWERS: 1) (5.5, 9) 2) 8.06

Knowledge Organiser: 19a
Types of Graph

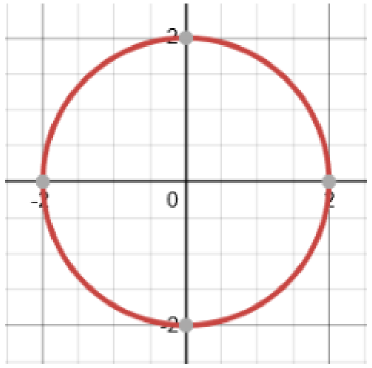
Examples



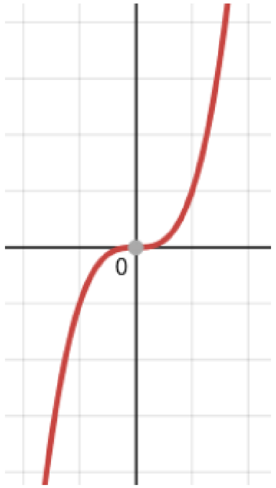
Quadratic graphs
$y = x^2$



Reciprocal graphs
$$y = \frac{1}{x}$$



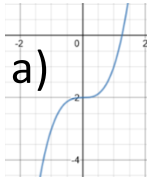
Circle graphs
$x^2 + y^2 = 4$

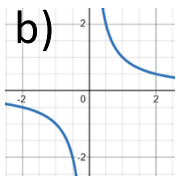


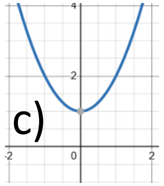
Cubic graphs
$y = x^3$

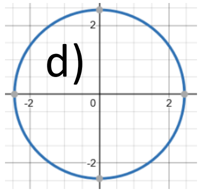
Key Words
Quadratic
Cubic
Reciprocal
Circle
Graph

Match the graph with the correct equation:









1) $x^2 + y^2 = 6$
2) $y = \frac{1}{x}$
3) $y = x^3 - 2$
4) $y = x^2 + 1$

ANSWERS: 1d 2b 3a 4c

Knowledge Organiser: 19b
Direct and Inverse Proportion

Key Concepts

Variables are **directly proportional** when the **ratio is constant** between the quantities.

Variables are **inversely proportional** when **one quantity increases in proportion to the other decreasing.**

Examples

Direct proportion:

Value of A	32	P	56	20	72
Value of B	20	30	35	R	45

Ratio constant: $20 \div 32 = \frac{5}{8}$

From A to B we will multiply by $\frac{5}{8}$.

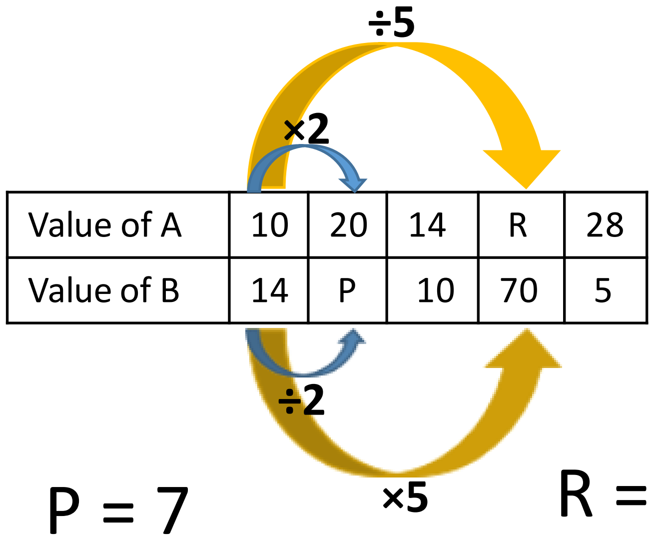
From B to A we will divide by $\frac{5}{8}$.

$P = 30 \div \frac{5}{8} = 48$

R= 20

$\times \frac{5}{8} = 12.5$

Inverse proportion:



Value of A	10	20	14	R	28
Value of B	14	P	10	70	5

P = 7

R = 2

Key Words

Direct
Inverse
Proportion
Divide
Multiply
Constant

Complete each table:

1) Direct proportion

Value of A	5	P	22
Value of B	9	28.8	Q

2) Inverse proportion

Value of A	4	P	18
Value of B	9	3	Q

ANSWERS 1) P = 16, Q = 39.6 2) P = 12, Q = 2

Knowledge Organiser: 19b
Direct and Inverse Proportion Using Algebra

Key Concepts

Variables are **directly proportional** when the **ratio is constant** between the quantities.

Variables are **inversely proportional** when **one quantity increases in proportion to the other decreasing.**

α is the symbol we use to show that one variable is in proportion to another.

Direct proportion: $y \propto x$

Inverse proportion: $y \propto \frac{1}{x}$

Examples

Direct proportion:

g is directly proportional to the square root of *h*
When *g* = 18, *h* = 16
Find the possible values of *h* when *g* = 2

$$g \propto \sqrt{h}$$
$$g = k\sqrt{h}$$
$$18 = k\sqrt{16}$$
$$18 = 4k$$
$$4.5 = k$$
$$g = 4.5\sqrt{h}$$

$$g = 4.5\sqrt{h}$$
When $g = 2$
$$2 = 4.5\sqrt{h}$$
$$\frac{2}{4.5} = \sqrt{h}$$
$$\left(\frac{4}{9}\right)^2 = h$$
$$\frac{16}{81} = h$$

Inverse proportion:

The time taken, t, for passengers to be checked-in is inversely proportional to the square of the number of staff, s, working.
It takes 30 minutes for passengers to be checked-in when 10 staff are working. How many staff are needed for 120 minutes?

$$t \propto \frac{1}{s^2}$$
$$t = \frac{k}{s^2}$$
$$30 = \frac{k}{10^2}$$
$$3000 = k$$
$$t = \frac{3000}{s^2}$$

$$t = \frac{3000}{s^2}$$
$$120 = \frac{3000}{s^2}$$
$$s^2 = \frac{3000}{120}$$
$$s^2 = 25$$
$$s = \sqrt{25}$$
$$s = 5$$

Key Words

Direct
Inverse
Proportion
Divide
Multiply
Constant

1) e is directly proportional to f
When $e = 3$, $f = 36$
Find the value of f when $e = 4$

2) x is inversely proportional to the square root of y.
When $x = 12$, $y = 9$
Find the value of x when $y = 81$

ANSWERS 1) $f = 48$ 2) $x = 4$

Knowledge Organiser: 19b
Direct and Inverse Proportion on Graphs

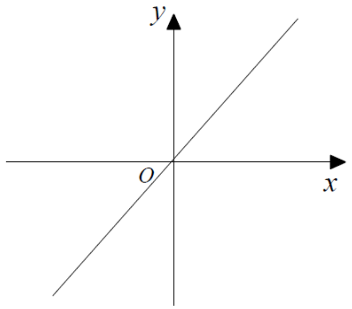
Key Concepts

Variables are **directly proportional** when the **ratio is constant** between the quantities.

Variables are **inversely proportional** when **one quantity increases in proportion to the other decreasing.**

Direct and inverse proportion can also be represented on **graphs**.

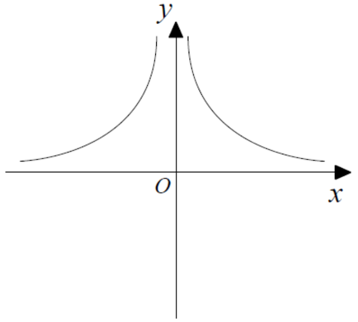
Examples



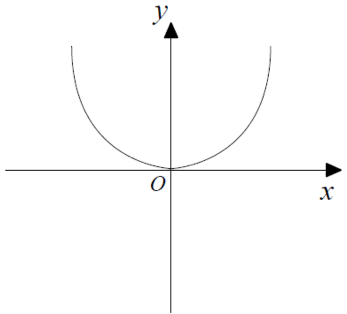
$y \propto x$

y is directly proportional to x

$y \propto \frac{1}{x}$



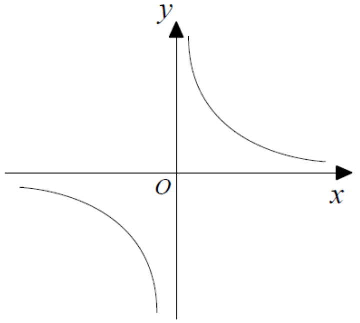
y is inversely proportional to x



$y \propto x^2$

y is directly proportional to x^2

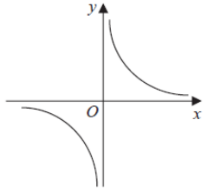
$y \propto \frac{1}{x^2}$



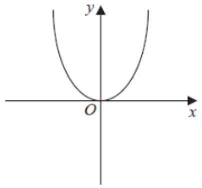
y is inversely proportional to x^2

Key Words

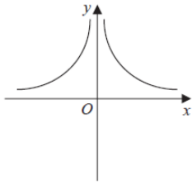
Direct
Inverse
Proportion
Graph



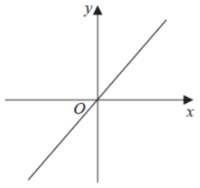
Graph A



Graph B



Graph C



Graph D

Match the correct graph to each statement:

Proportionality relationship	Graph letter
y is directly proportional to x	
y is inversely proportional to x	
y is proportional to the square of x	
y is inversely proportional to the square of x	

ANSWERS D, A, B, C